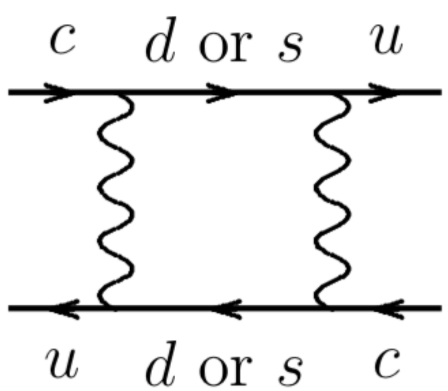
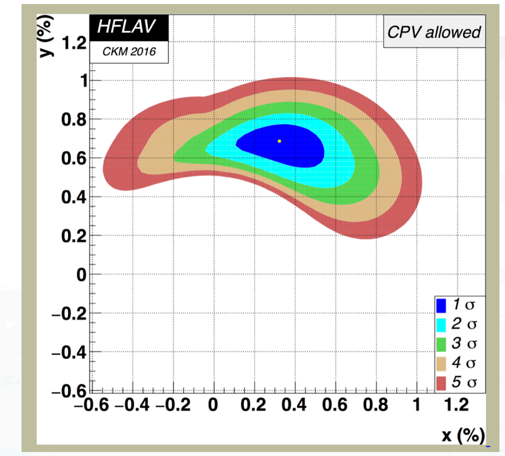
New Physics in the Charm Sector





Alexander Lenz, IPPP Durham

23./24. August 2018

The 2018 Weihai High-Energy Physics School

Outline

1. **Indirect searches for new physics**
2. **Heavy Quark Expansion**
3. **D-Mixing**
4. **Rare decays**

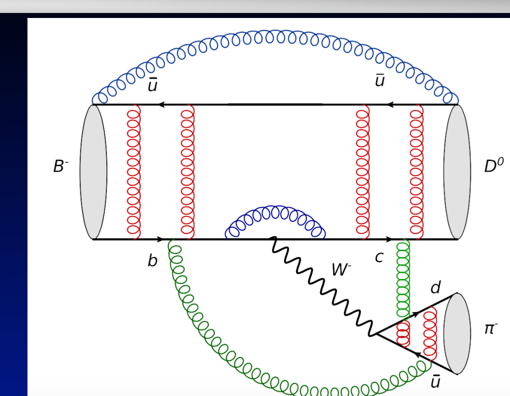
MOTIVATION FOR FLAVOUR PHYSICS

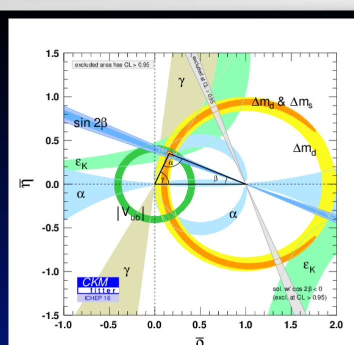
Baryon Asymmetry in the Universe:
A violation of the CP symmetry - which causes matter and anti-matter to evolve differently with time - seems to be necessary to explain the existence of matter in the Universe.
CP violation has so far only been found in hadron decays, which are experimentally investigated at LHCb and NA62 (CERN), SuperBelle (Japan),…

Indirect Search for BSM Physics:
To find hints for Physics beyond the Standard Model we can either use brute force (= higher energies) or more subtle strategies like high precision measurements.
New contributions to an observable f are identified via:

$$f^{\mathrm{SM}} + f^{\mathrm{NP}} = f^{\mathrm{Exp}}$$

Understanding QCD:
Hadron decays are strongly affected by QCD (strong interactions) effects, which tend to overshadow the interesting fundamental decay dynamics. Theory tools like effective theories, Heavy Quark Expansion, HQET, SCET ,…enable a control over QCD-effects and they are used in other fields like Collider Physics, Higgs Physics, DM searches…





Standard Model parameters:
Hadron decays depend strongly on Standard Model parameters like quark masses and CKM couplings (which are the only known source of CP violation in the SM). A precise knowledge of these parameters is needed for all branches of particle physics.

E.g. lecture by A. Soni

ANOMALIES 2 - 6 σ

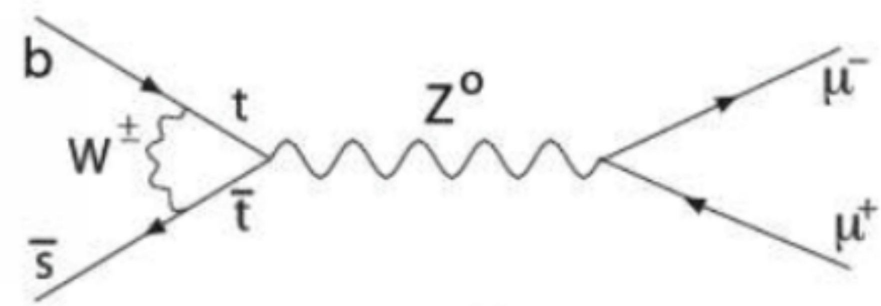
- **There are interesting anomalies in Flavour Physics**

  - 3-6: Semi-leptonic loop-level decays
  - 3.9: Semi-leptonic tree-level decays
  - 3.6: B-mixing phase (dimuon)
  - 3.5: CPV in Kaons (huge lattice progress)
  - 2.6: 30 GeV resonance (ALEPH)
  - 2.6: Zbb coupling (LEP FB asym)
  - 2.x: K-pi puzzle
  - 2.0: B-mixing modulus (mass difference)

4 σ in neutron lifetime? Proton radius seems to be solved by Hänsch et al

SEMI-LEPTONIC LOOP LEVEL

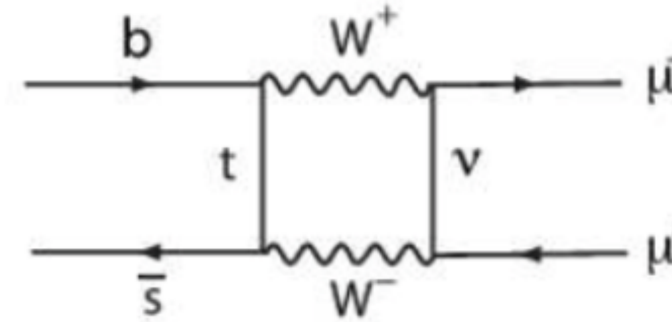
$$b \to s\mu\mu$$



relatively simple hadronic structure

$B_{d,s} \to \mu\mu$: **decay constant**

$H_b \to H_q\mu\mu$: **form factor**



Can be determined with lattice, sum rules,…

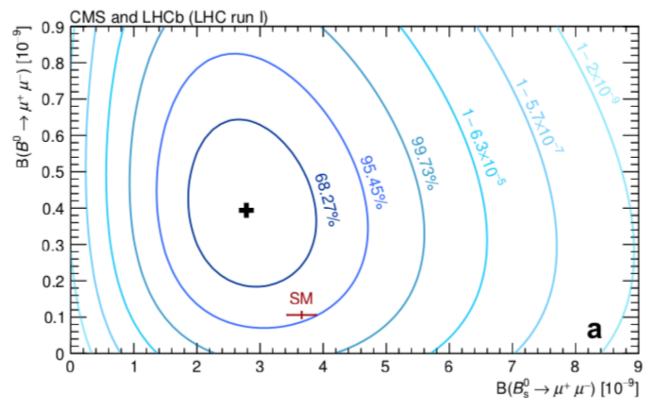
Observables:

- Branching ratios $Br(B_s \to \phi\mu\mu),\ Br(B \to K^*\mu\mu),$
- Angular observables, e.g. P_5' **hadronic uncertainties cancel partially**
- Ratios $R_K = \frac{Br(B^+ \to K^+\mu^-\mu^+)}{Br(B^+ \to K^+e^-e^+)}$ **hadronic uncertainties cancel almost completely**

Understanding hadronic uncertainties is crucial for identifying BSM effects!

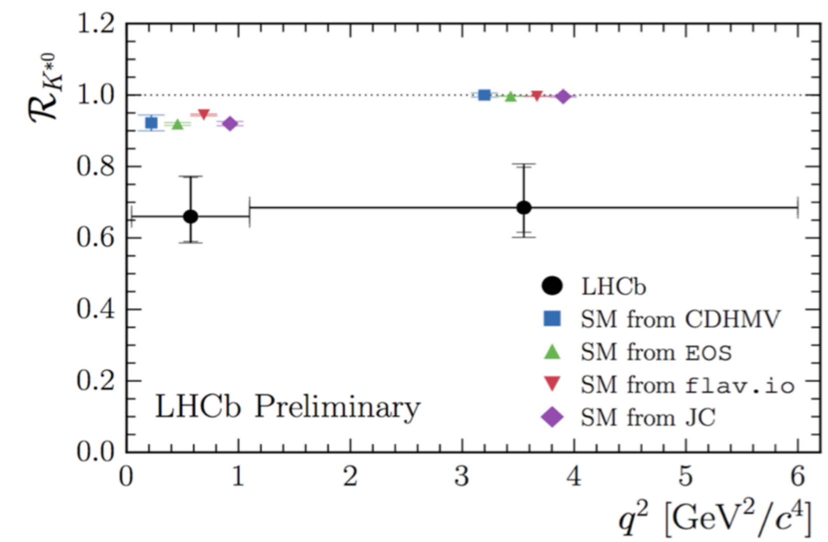
SEMI-LEPTONIC LOOP LEVEL $b \to s\mu\mu$

$B_{d,s} \to \mu\mu$

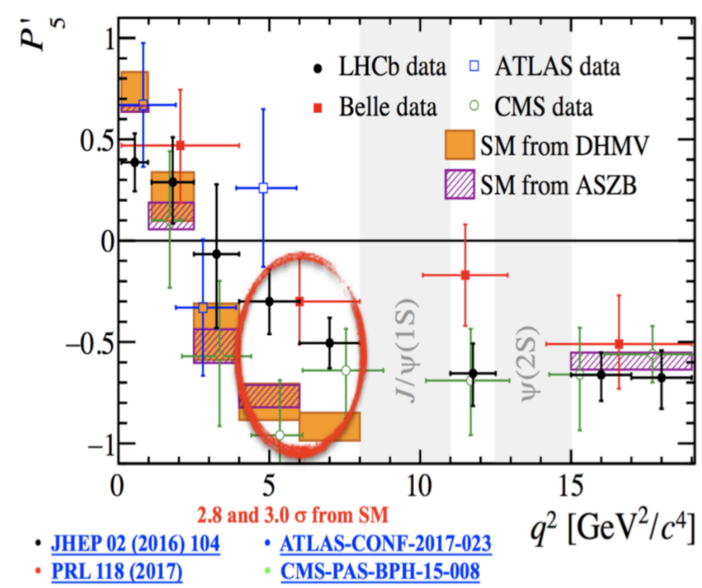


$H_b \to H_q\mu\mu$

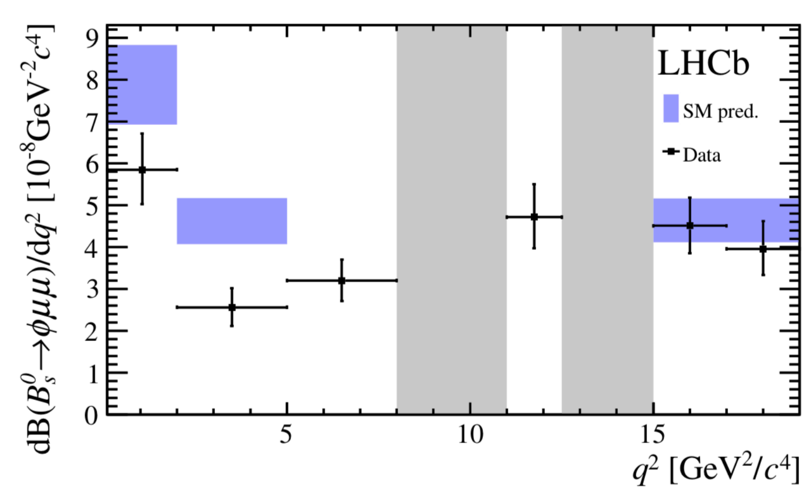
a) $R_K = \dfrac{Br(B^+ \to K^+\mu^-\mu^+)}{Br(B^+ \to K^+e^-e^+)}$



b) P_5'



c) $Br(B_s \to \phi\mu\mu)$, $Br(B \to K^*\mu\mu)$



SEMI-LEPTONIC LOOP LEVEL

Consistent picture of numerous (175) observables

all can be fitted in very simple scenario (BSM = -1/4 SM)

$$Q_{9V} = \frac{\alpha_e}{4\pi}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{l}\gamma^\mu l\right)$$

$$Q_{10A} = \frac{\alpha_e}{4\pi}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{l}\gamma^\mu\gamma^5 l\right)$$

e.g. just modify the Wilson coefficient C9!

3 σ 1704.05447

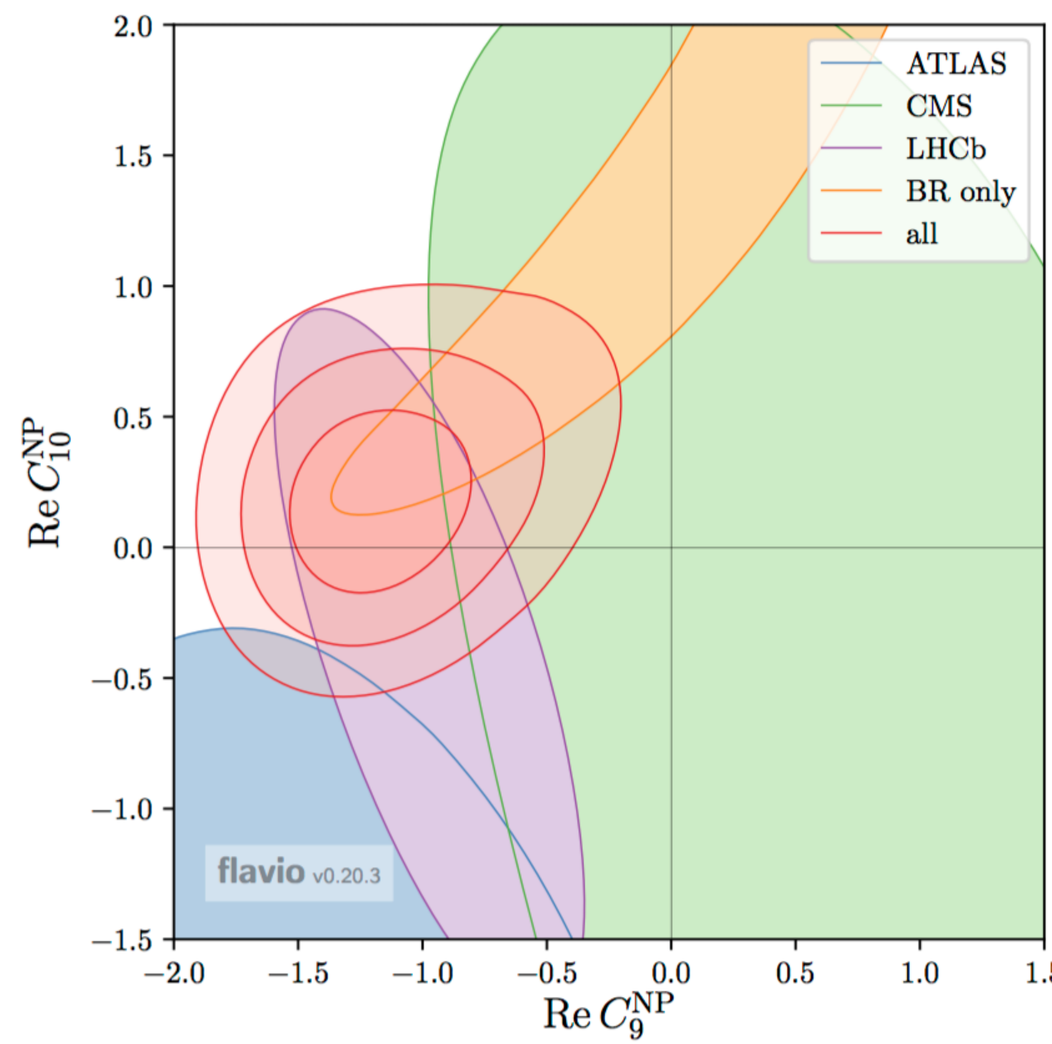
Ciuchini, Coutinho, Fedele, Franco, Paul, Silvestrini, Valli

On Flavourful Easter eggs for NP hunger and LFU violation

5.7 σ 1704.05340

Capdevilla, Cvrivellin, Descotes-Genon, Matias, Virto

Patterns of NP in b to all transitions in the light of recent data

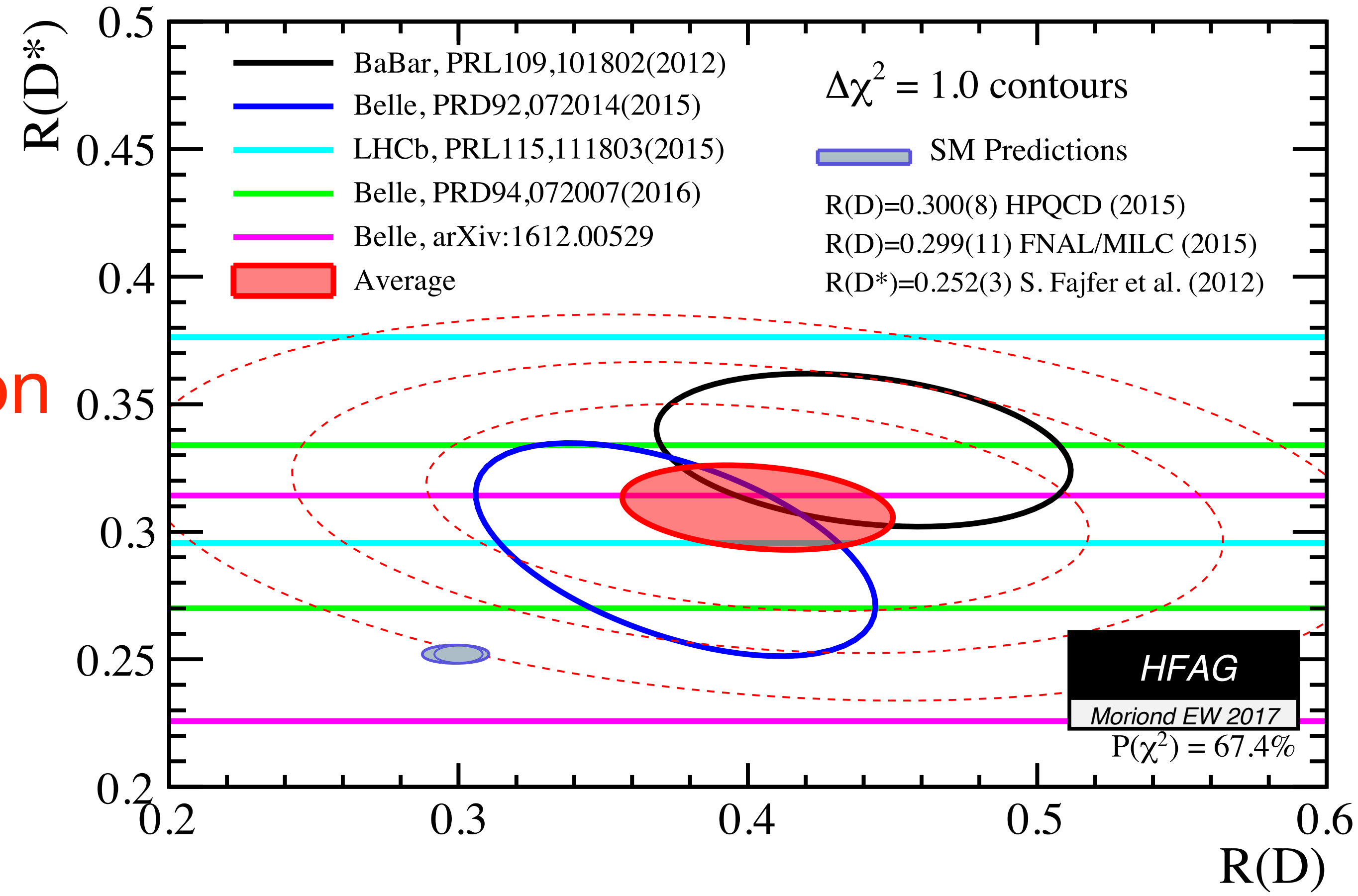


arXiv:1703.09189 [pdf, other]
Status of the $B \to K^*\mu^+\mu^-$ anomaly after Moriond 2017
Wolfgang Altmannshofer, Christoph Niehoff, Peter Stangl, David M. Straub

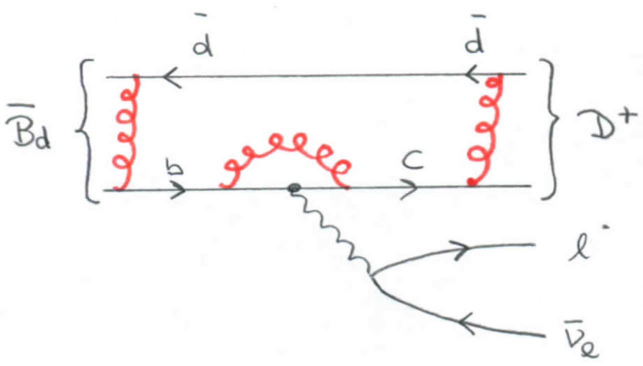
Statistical significance depends on estimate of hadronic uncertainties!

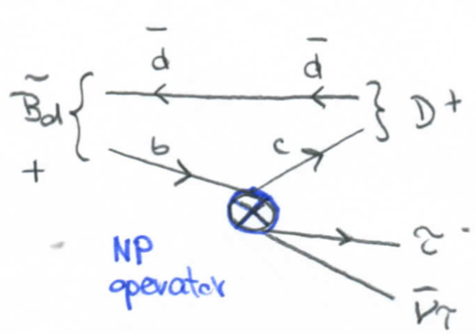
SEMI-LEPTONIC TREE LEVEL (THIS IS LARGE!)

$$R_{D^{(*)}} = \frac{Br(\bar{B} \to D^{(*)}\tau^- \bar{\nu}_\tau)}{Br(\bar{B} \to D^{(*)} l^- \bar{\nu}_l)}$$

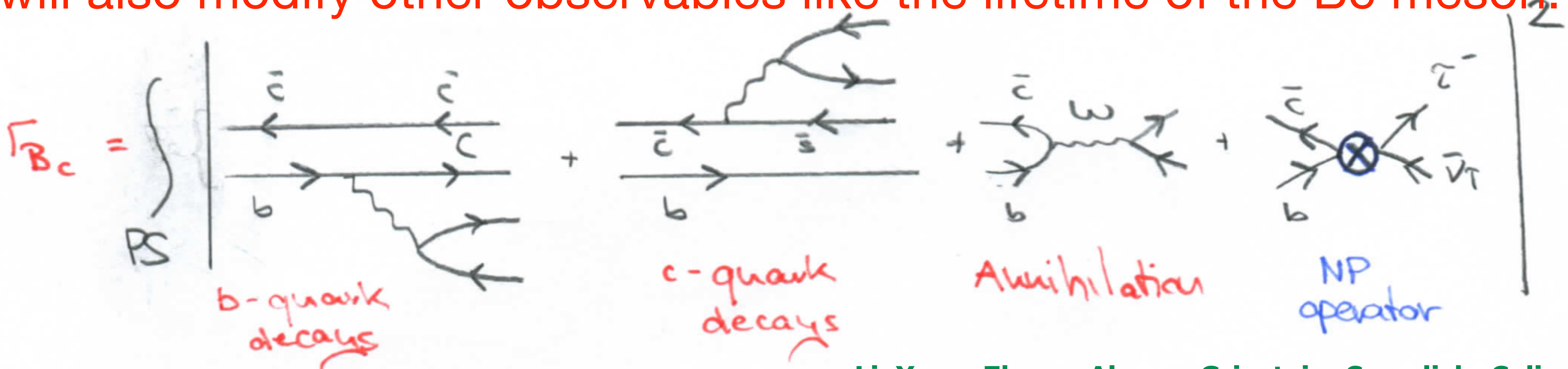


Beware: any new $b \to c\tau\bar{\nu}_\tau$ contribution





will also modify other observables like the lifetime of the Bc meson!



e.g. Li, Yang, Zhang; Alonso, Grinstein, Camalich; Celis, Jung, Li, Pich

ASSUME ANOMALIES ARE REAL

Incomplete list of models:

- **Z' - new U(1) or SU(2)**
- **Leptoquarks**
- W'- new SU(2)
- Composite Models
- WED
- SUSY
- 2HDM
- ….
- ….

"Qual der Wahl" oder "Wahl der Qual"

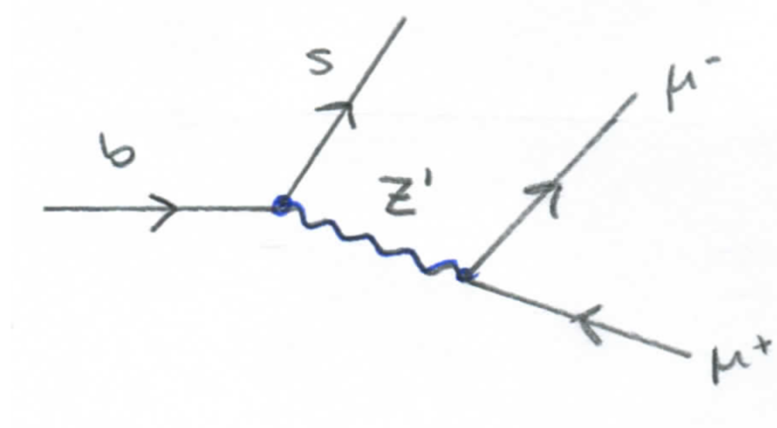
=

agony of choice or choice of agony?

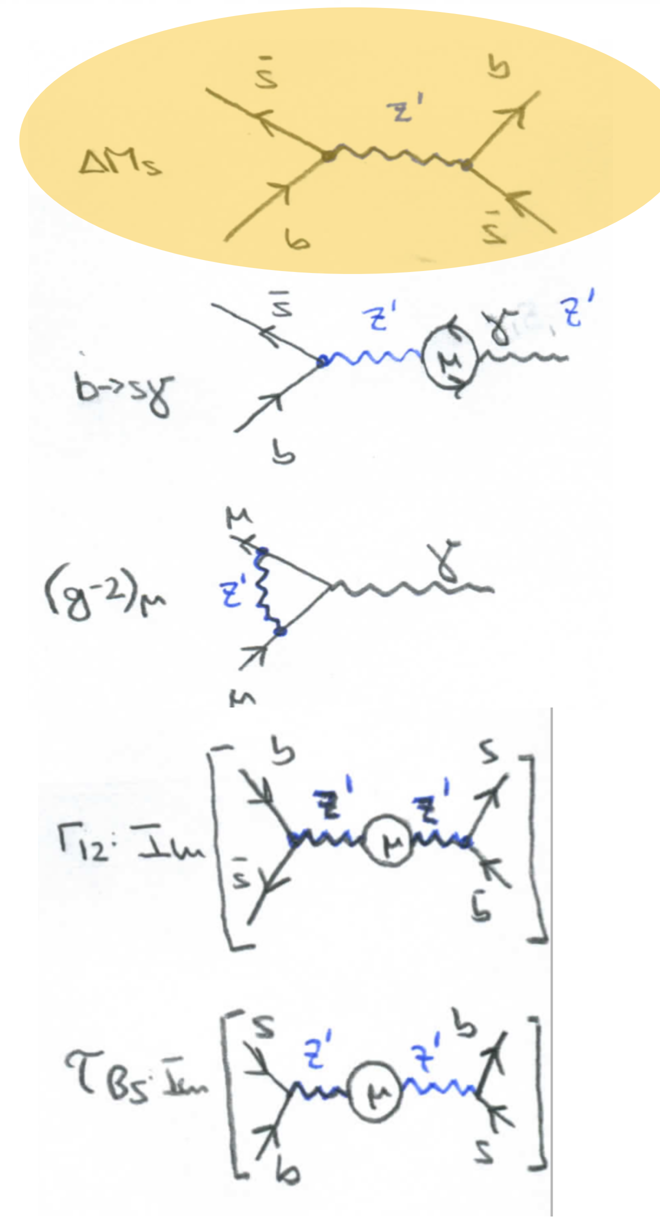
hundreds of papers…

ASSUME ANOMALIES ARE REAL

A popular BSM model for solving the anomalies related to loop-level (semi) leptonic decays are Z' models:



Such a new tree-level transition will also affect many other observables, most notably **B-mixing at tree-level**, but also many loop processes.



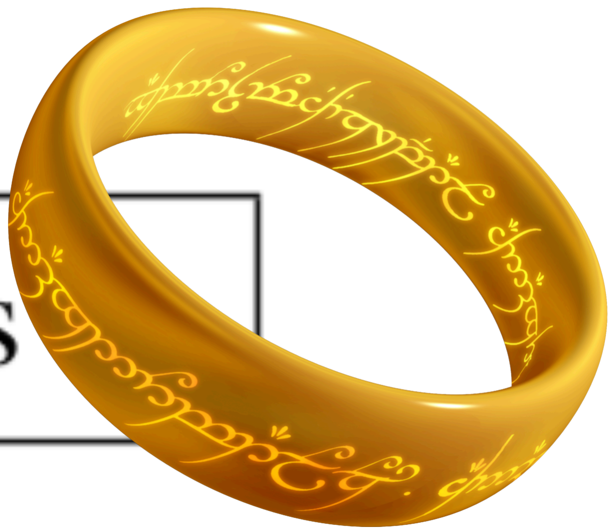
Make sure all relevant bounds are included, e.g. electro-weak precision bounds

1705.00929 Ferruccio Feruglio, Paride Paradisi, Andrea Pattori

On the Importance of Electroweak Corrections for B Anomalies

NEW: Bs mixing "disagrees" with the SM

using most recent input, in particular most recent lattice values for the hadronic input
fBs^2 B two Bs mixing
from FLAG (dominated by Fermilab/MILC)



$$\boxed{\Delta M_s^{\mathrm{SM},\,2017} = (20.01 \pm 1.25)\ \mathrm{ps}}$$

$$\Delta M_s^{\mathrm{Exp}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$

BSM contributions should be **negative**
very stringent bound on many BSM models that explain the b-> s mu mu anomalies

$$\frac{\Delta M_s^{\mathrm{Exp}}}{\Delta M_s^{\mathrm{SM}}} = \left| 1 + \frac{\kappa}{\Lambda_{\mathrm{NP}}^2} \right|$$

$$\frac{\Lambda_{\mathrm{NP}}^{2017}}{\Lambda_{\mathrm{NP}}^{2015}} = \sqrt{\frac{\frac{\Delta M_s^{\mathrm{Exp}}}{\left(\Delta M_s^{\mathrm{SM}} - 2\delta \Delta M_s^{\mathrm{SM}}\right)^{2015}} - 1}{\frac{\Delta M_s^{\mathrm{Exp}}}{\left(\Delta M_s^{\mathrm{SM}} - 2\delta \Delta M_s^{\mathrm{SM}}\right)^{2017}} - 1}} \simeq 5.2$$

One constraint to kill them all?

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$$\frac{\Delta M_s^{\rm Exp}}{\Delta M_s^{\rm SM}} = \left|1 + \frac{C_{bs}^{LL}}{R_{\rm SM}^{\rm loop}}\right|$$

Abstract

Many BSM models that explain the intriguing anomalies in the quark flavour sector are severely constrained by B_s-mixing, for which the SM prediction and experiment agreed well until recently. New non-perturbative calculations point, however, in the direction of a tiny discrepancy in this observable. Using this new input we find a considerable shift of the bounds on BSM models stemming from B_s-mixing.

$$C_{bs}^{LL} = \frac{\eta^{LL}(M_{Z'})}{4\sqrt{2}G_F M_{Z'}^2}\left(\frac{\lambda_{23}^Q}{V_{tb}V_{ts}^*}\right)^2$$

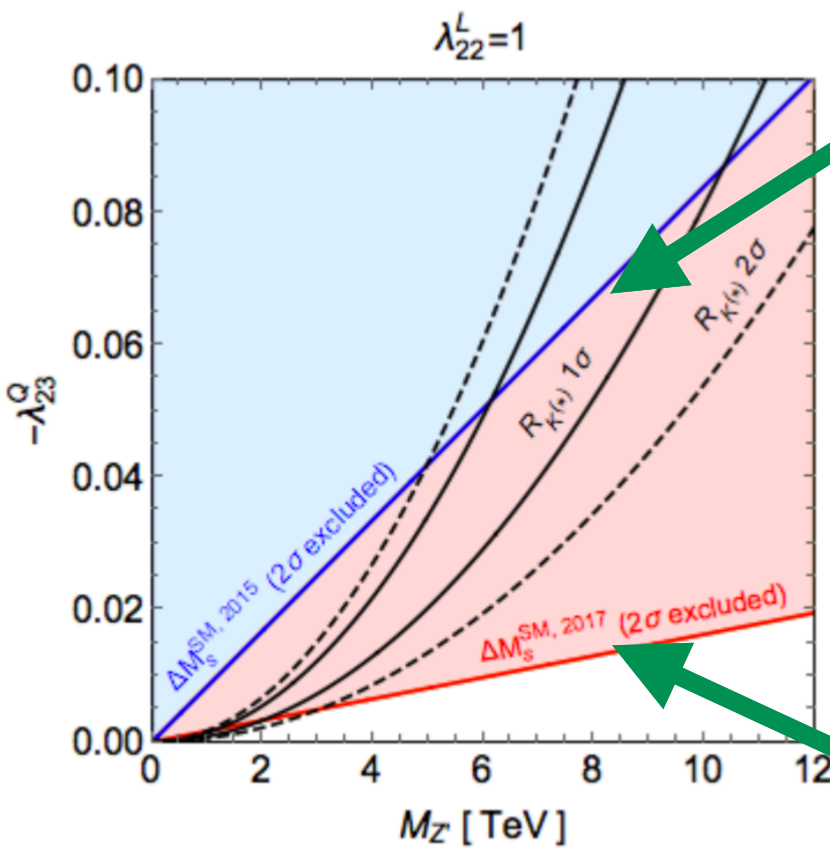


FIG. 2. Bounds from B_s-mixing on the parameter space of the simplified Z' model of Eq. (20), for real λ_{23}^Q and $\lambda_{22}^L = 1$. The blue and red shaded areas correspond respectively to the 2σ exclusions from $\Delta M_s^{\rm SM,\,2015}$ and $\Delta M_s^{\rm SM,\,2017}$, while the solid (dashed) black curves encompass the 1σ (2σ) best-fit region from $R_{K^{(*)}}$.

FNAL/MILC

Previous lattice results

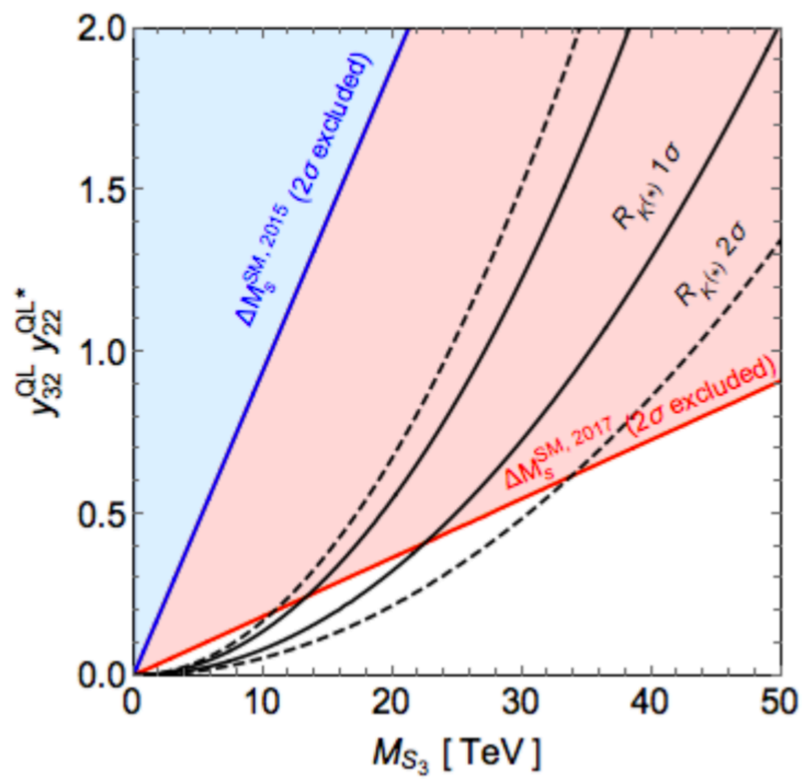


FIG. 3. Bounds from B_s-mixing on the parameter space of the scalar leptoquark model of Eq. (24), for real $y_{32}^{QL} y_{22}^{QL*}$ couplings. Meaning of shaded areas and curves as in Fig. 2.

Hadronic SM effects decide whether a BSM models is excluded or not

INDIRECT SEARCHES FOR NEW PHYSICS

Leave no stone unturned: So far most anomalies in the b-sector
— charm physics is complementary! —-

Two lessons from recent history of Delta A_CP

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+ K^-) - A_{CP}(D^0 \to \pi^+ \pi^-)$$

CHARM 2013 Manchester

- Ancient knowledge:

There is no CPV in the charm system

- **LHCb: 1112.0938 - 175 citations**

There is CPV in the charm system

- Theoretical (re)considerations

This is a clear indication of NP

NP = New Physics vs. NP = Non-perturbative QCD

- **LHCb: 1303.2614 - 15 citations**

What we actually meant: there is no CPV in the charm system

- Theorists: **Experimentalists have to work harder!**

INDIRECT SEARCHES FOR NEW PHYSICS

To identify BSM effects from indirect searches:

hadronic SM effects have to be understood

To study constraints on potential BSM models explaining the anomalies:

hadronic SM effects have to be understood

Charm physics is complementary

Lessons from Delta A_CP: Experimental numbers can change significantly

We do not really understand the SM contributions to charm physics

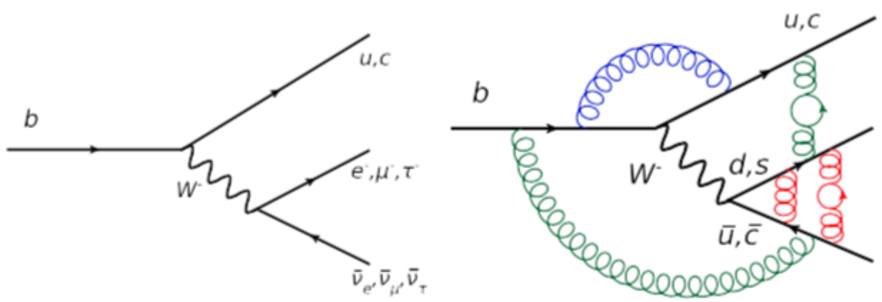
Try to understand SM contributions to Charm Physics

HEAVY QUARK EXPANSION

1). The effective Hamiltonian

OPE 1: OBVIOUS START

Effective Hamiltonian: Why not simply calculate in the full Standard Model?



Calculating the total inclusive decay rate of a b-quark we get

$$\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 c_{3,b}\,, \qquad (87)$$

with

$$\begin{aligned} c_{3,b} = & \left\{ g\left(\frac{m_c}{m_b},\frac{m_e}{m_b}\right) + g\left(\frac{m_c}{m_b},\frac{m_\mu}{m_b}\right) + g\left(\frac{m_c}{m_b},\frac{m_\tau}{m_b}\right) \right. \\ & + N_c|V_{ud}|^2 h\left(\frac{m_c}{m_b},\frac{m_u}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{us}|^2 h\left(\frac{m_c}{m_b},\frac{m_u}{m_b},\frac{m_s}{m_b}\right) \\ & \left. + N_c|V_{cd}|^2 h\left(\frac{m_c}{m_b},\frac{m_c}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{cs}|^2 h\left(\frac{m_c}{m_b},\frac{m_c}{m_b},\frac{m_s}{m_b}\right) \right\} \\ & + \left|\frac{V_{ub}}{V_{cb}}\right|^2 \left\{ g\left(\frac{m_u}{m_b},\frac{m_e}{m_b}\right) + g\left(\frac{m_u}{m_b},\frac{m_\mu}{m_b}\right) + g\left(\frac{m_u}{m_b},\frac{m_\tau}{m_b}\right) \right. \\ & + N_c|V_{ud}|^2 h\left(\frac{m_u}{m_b},\frac{m_u}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{us}|^2 h\left(\frac{m_u}{m_b},\frac{m_u}{m_b},\frac{m_s}{m_b}\right) \\ & \left. + N_c|V_{cd}|^2 h\left(\frac{m_u}{m_b},\frac{m_c}{m_b},\frac{m_d}{m_b}\right) + N_c|V_{cs}|^2 h\left(\frac{m_u}{m_b},\frac{m_c}{m_b},\frac{m_s}{m_b}\right) \right\}. \end{aligned} \qquad (88)$$

Phase space

$$f\left(\frac{m_c}{m_b}\right) = \begin{cases} 0.484 \\ 0.518 \\ 0.666 \end{cases} \text{for} \begin{cases} m_c^{\rm Pole} = 1.471\ \text{GeV}, & m_b^{\rm Pole} = 4.650\ \text{GeV} \\ \bar m_c(\bar m_c) = 1.277\ \text{GeV}, & \bar m_b(\bar m_b) = 4.248\ \text{GeV} \\ \bar m_c(\bar m_b) = 0.997\ \text{GeV}, & \bar m_b(\bar m_b) = 4.248\ \text{GeV} \end{cases} \qquad (90)$$

$$c_{3,b} = \begin{cases} 9 \\ 2.97 \\ 3.25 \\ 4.66 \end{cases} \text{for} \begin{cases} m_c = 0, \\ m_c^{\rm Pole}, m_b^{\rm Pole} \\ \bar m_c(\bar m_c), \bar m_b(\bar m_b) \\ \bar m_c(\bar m_b), \bar m_b(\bar m_b) \end{cases}.$$

Overall result including NLO-QCD

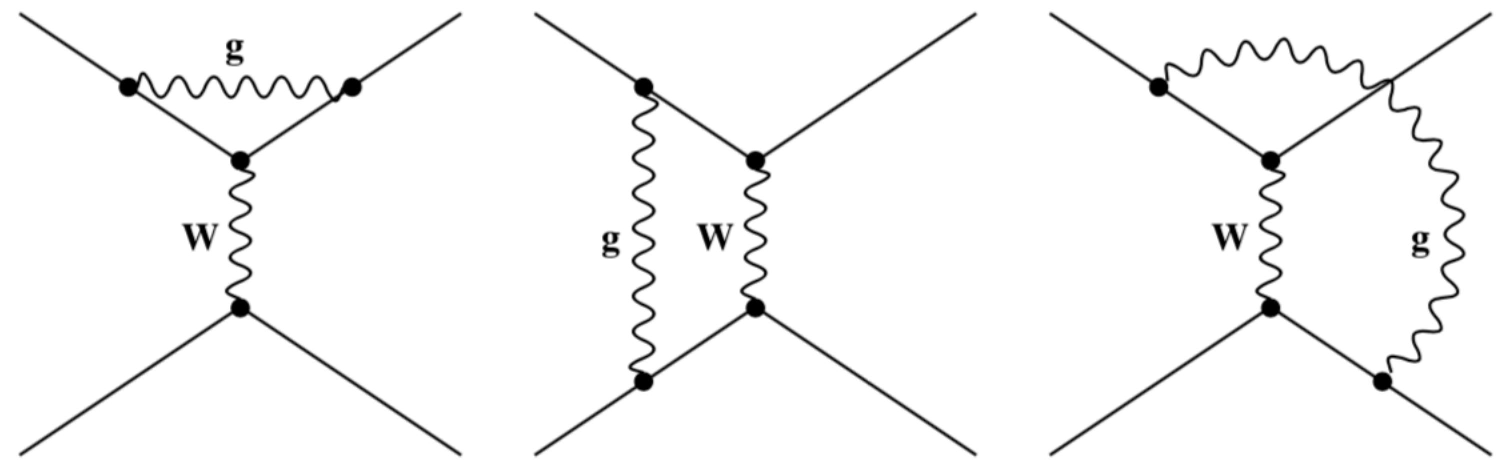
$$\tau_b = 2.60\ \text{ps} \quad \text{for} \quad \bar m_c(\bar m_b),\ \bar m_b(\bar m_b)\ .$$

***b*-hadron species**	**average lifetime**	**lifetime ratio**
B^0	**1.518 ± 0.004 ps**	
B^+	**1.638 ± 0.004 ps**	$B^+/B^0 = 1.076 \pm 0.004$
B_s^0	**1.509 ± 0.004 ps**	$B_s^0/B^0 = 0.994 \pm 0.004$

This is not satisfactory!

OPE 2: PROBLEM

Calculating QCD corrections in the full Standard Model



we find an expansion in $\alpha_s \ln\left(\frac{m_b^2}{M_W^2}\right) \approx 6\alpha_s$ approx 1.2 and not in alpha_s!

Tree	1	$-$	$-$	$-$
1-loop	$\alpha_s \ln$	α_s	$-$	$-$
2-Loop	$\alpha_s^2 \ln^2$	$\alpha_s^2 \ln$	α_s^2	$-$
3-loop	$\alpha_s^3 \ln^3$	$\alpha_s^3 \ln^2$	$\alpha_s^3 \ln$	α_s^3
	$\dots$	$\dots$	$\dots$	$\dots$

Series does not converge at all!

OPE 3: SOLUTION

Formal definition:

The most important tool in the analysis of operator products in quantum field theory is Wilson's *operator product expansion* (OPE) [5], which states that any product of local quantum fields can be expanded as

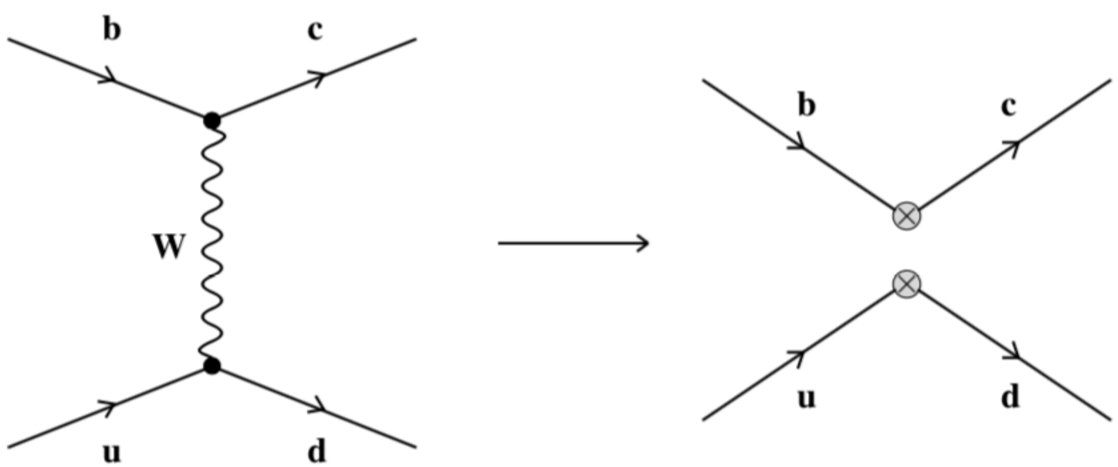
$$\mathcal{O}_{A_1}(x_1) \cdots \mathcal{O}_{A_N}(x_N) \sim \sum_B \mathcal{C}^B_{A_1 \ldots A_N}(x_1, \ldots, x_N)\, \mathcal{O}_B(x_N)\,. \tag{1.0.1}$$

Here the symbols $\mathcal{O}_A$ denote the composite fields that appear in the given theory, where the label A also incorporates the tensor or spinor character of the field. The so called OPE coefficients, $\mathcal{C}^B_{A_1 \ldots A_N}$, are distributions with singularities on the diagonals $x_i = x_j$

probably not too helpful

OPE 4: SOLUTION

Real life definition: integrate out the heavy particles



l.h.s.

Operator 1 = bc quark current

Operator 2 = ud quark current

r.h.s.

Wilson coefficient = Fermi constant

4 quark operator

$$\frac{ig_2 V_{cb}^*}{2\sqrt{2}} \frac{1}{k^2 - M_W^2} \frac{ig_2 V_{ud}}{2\sqrt{2}} \approx \left(\frac{g_2}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} V_{CKM} =: \frac{G_F}{\sqrt{2}} V_{CKM}$$

=> Effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3}^{6} C_j Q_j \right] .$$

$$\begin{aligned} Q_2 &= (\bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha) \times (\bar{d}_\beta \gamma^\mu (1-\gamma_5) u_\beta) \\ &=: (\bar{c}_\alpha b_\alpha)_{V-A} \times (\bar{d}_\beta u_\beta)_{V-A} , \end{aligned}$$

hopefully helpful

OPE 5: SOLUTION

=> Effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3}^{6} C_j Q_j \right] .$$

No QCD corrections: C_1 = 0, C_2 = 1

With QCD corrections and Renormalistion Group Evolution:

1. Numerical correction: C_1 = -0.2, C_2 = 1.1

2. Separation of scales: short distance in Wilson coefficients (e.g. M_W, but also BSM)
long distance in matrix elements of operators

3. Summation of large logs to all orders!!!

Heff

SM

	LL	NLL	NNLO	NNNLO
Tree	1	–	–	–
1-loop	$\alpha_s \ln$	α_s	–	–
2-Loop	$\alpha_s^2 \ln^2$	$\alpha_s^2 \ln$	α_s^2	–
3-loop	$\alpha_s^3 \ln^3$	$\alpha_s^3 \ln^2$	$\alpha_s^3 \ln$	α_s^3
	...	...	...	...

hopefully helpful

OPE 6: PHENOMENOLOGY

The effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3}^{6} C_j Q_j \right] .$$

is used for calculating observables

1. Total decay rates = 1/lifetime - Heavy Quark Expansion (HQE)

2. Mixing observables: decay rate difference (HQE)

3. Rare decays: b => s gamma (C_7), B to K(*) mu mu (C_9); Bs to mu mu (C_10)

HEAVY QUARK EXPANSION

2. Total decay rates - lifetimes

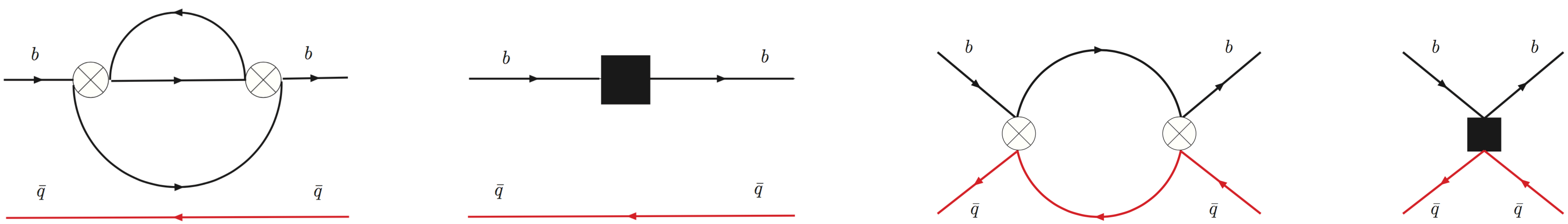
HEAVY QUARK EXPANSION - LIFETIMES

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X \int_{\mathrm{PS}} (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X | \mathcal{H}_{eff} | B \rangle|^2$$

Assume:

- mb is large compared to hadronic scale
- decay rate is a Taylor series in 1/mb

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[c_{3,b} \frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2} \frac{\langle B|\bar{b} g_s \sigma_{\mu\nu} G^{\mu\nu} b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3} \frac{\langle B|(\bar{b}q)_\Gamma (\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right]$$



Remarks:

- leading term (=free quark decay) is universal
- different B mesons differ from the 3rd term on
- lifetime predictions need: non-perturbative matrix elements and perturbative Wilson coefficients

HEAVY QUARK EXPANSION

Total decay rate can be expanded in inverse powers of mb

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots$$

Each term in the series can be further expanded in the strong coupling

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \ldots$$

Each term is a product of a perturbative function and the matrix element of **Delta B = 0 operators** **(lattice , sum rules)**

Mixing obeys a similar HQE

$$\Gamma_{12}^q = \left(\frac{\Lambda}{m_b}\right)^3\Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4\Gamma_4 + \ldots$$

Now **Delta B = 2 operators appear** **(lattice , sum rules)**

STATUS BEFORE 2017

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$< \lvert \dim 6 \rvert >$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$< \lvert \dim 7 \rvert >$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

STATUS BEFORE 2017

$$\frac{\tau(B^+)}{\tau(B_d)}^{\text{HQE 2014}} = 1.04^{+0.07}_{-0.03}\,,$$

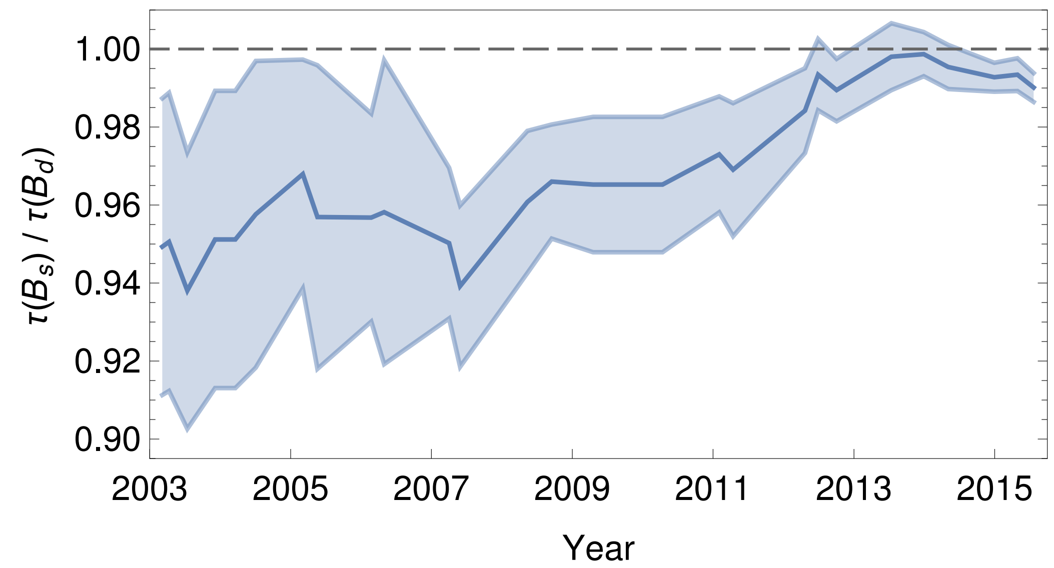
$$\frac{\tau(B_s)}{\tau(B_d)}^{\text{HQE 2014}} = 1.001 \pm 0.002\,,$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\text{HQE 2014}} = 0.935 \pm 0.054\,,$$

$$\frac{\bar\tau(\Xi_b^0)}{\bar\tau(\Xi_b^+)}^{\text{HQE 2014}} = 0.95 \pm 0.06\,.$$

*Large uncertainties due to outdated non-perturbative input

*Perfect cancellation in Bs lifetime - test of NP models



see e.g. Jäger et al 1701.091883

Observable	SM – conservative	SM – aggressive	Experiment
ΔM_s	$(18.3 \pm 2.7)\ \text{ps}^{-1}$	$(20.11 \pm 1.37)\ \text{ps}^{-1}$	$(17.757 \pm 0.021)\ \text{ps}^{-1}$
$\Delta\Gamma_s$	$(0.088 \pm 0.020)\ \text{ps}^{-1}$	$(0.098 \pm 0.014)\ \text{ps}^{-1}$	$(0.082 \pm 0.006)\ \text{ps}^{-1}$
a_{sl}^s	$(2.22 \pm 0.27) \cdot 10^{-5}$	$(2.27 \pm 0.25) \cdot 10^{-5}$	$(-7.5 \pm 4.1) \cdot 10^{-3}$

Ideal for NP searches - experimental precision > theory precision!

On the ultimate precision of meson mixing observables
Thomas Jubb, Matthew Kirk, Alexander Lenz, Gilberto Tetlalmatzi-Xolocotzi
Published in Nucl.Phys. B915 (2017) 431-453

NEWS

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$< \lvert\text{dim}\,6\rvert >$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$< \lvert\text{dim}\,7\rvert >$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

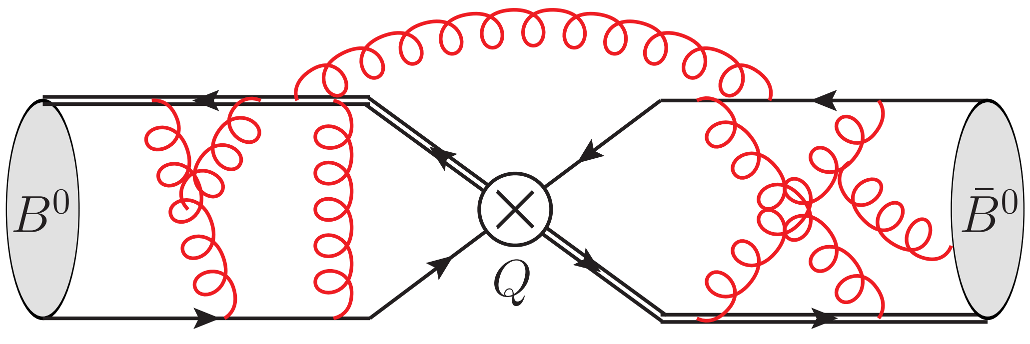
First steps: Asatrian et al 1709.02160

Sum rules: Kirk, Lenz, Rauh 1711.02100

HPQCD in progress, see LATTICE 2016, 2017

Sum rules: Kirk, Lenz, Rauh in progress

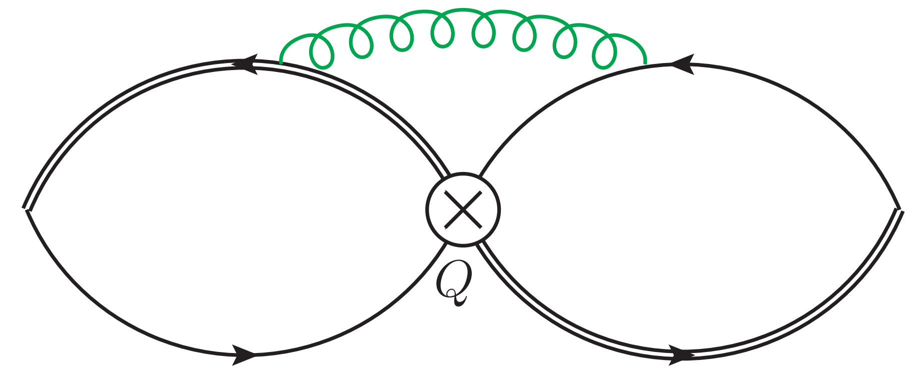
HQET SUM RULES



Sum rule

Quark-hadron duality

Analyticity



Hadronic matrix element

Characteristic scale: Λ_{QCD}

$\alpha_s\left(\Lambda_{\text{QCD}}\right) \sim \mathcal{O}(1)$

$\Rightarrow$ non-perturbative

Correlation function

Characteristic scale: 'virtuality' ω

Choose ω s.t. $\alpha_s(\omega) \ll 1$

$\Rightarrow$ perturbatively calculable

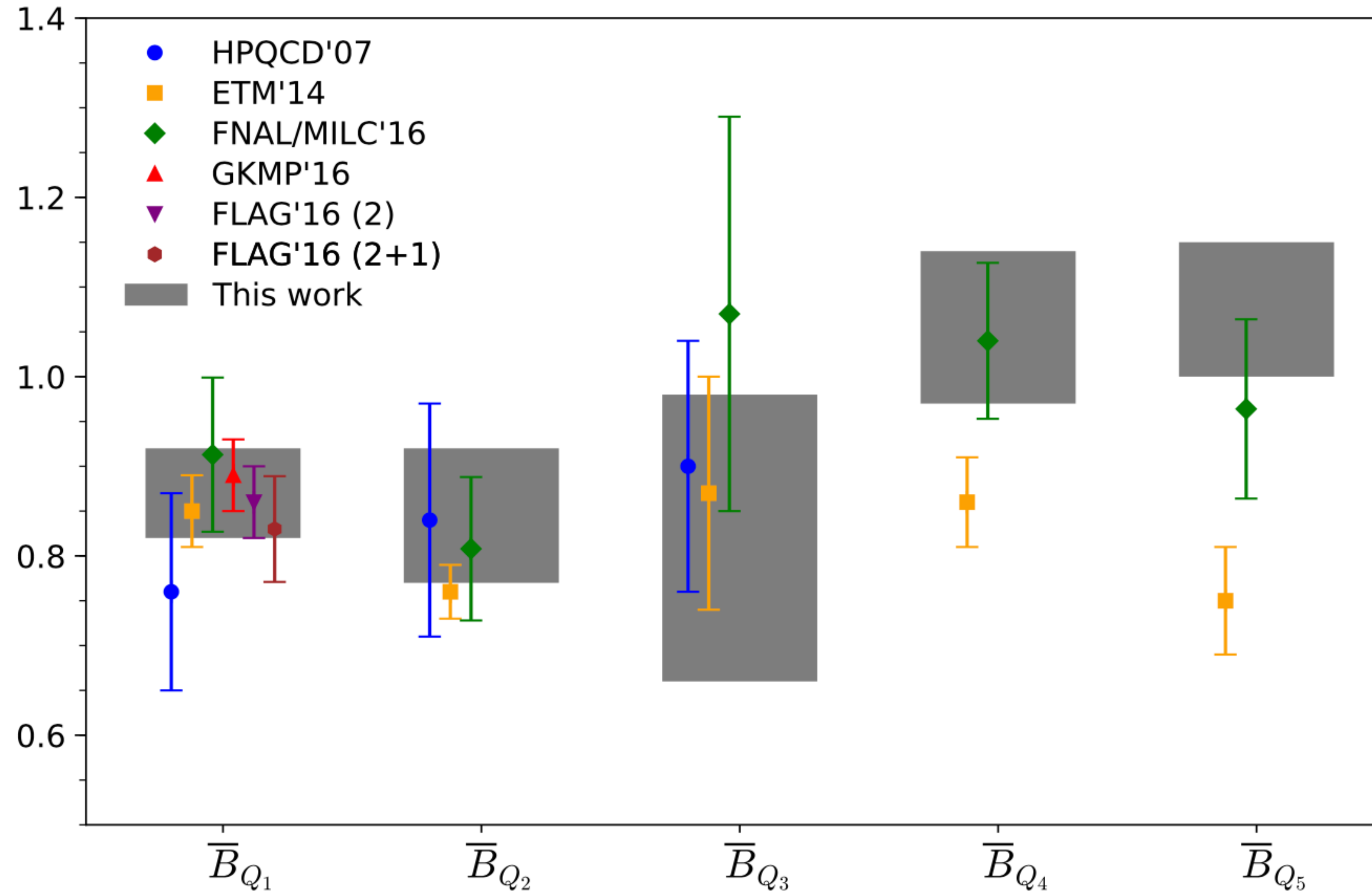
- Do all dim 6 and dim 7 operators for mixing **AND lifetimes**
- 3 loop diagrams with FIRE reduced (2 external momenta)
- Master integrals known: **Grozin, Lee; hep-ph/0812.4522**
- HQET running to scale m_b
- HQET-QCD matching at scale m_b

1 mixing operator Q done by Grozin, Klein, Mannel, Pivovarov hep-ph/1606.06054

all Delta B=0 and 2 dim 6 operators by Kirk, Lenz, Rauh; 1711.02100

NEW RESULTS 1: B-MIXING

Kirk, Lenz, Rauh 1711.02100



- **Nice agreement with lattice**
- **Comparable uncertainties as lattice -> halving error feasible!**

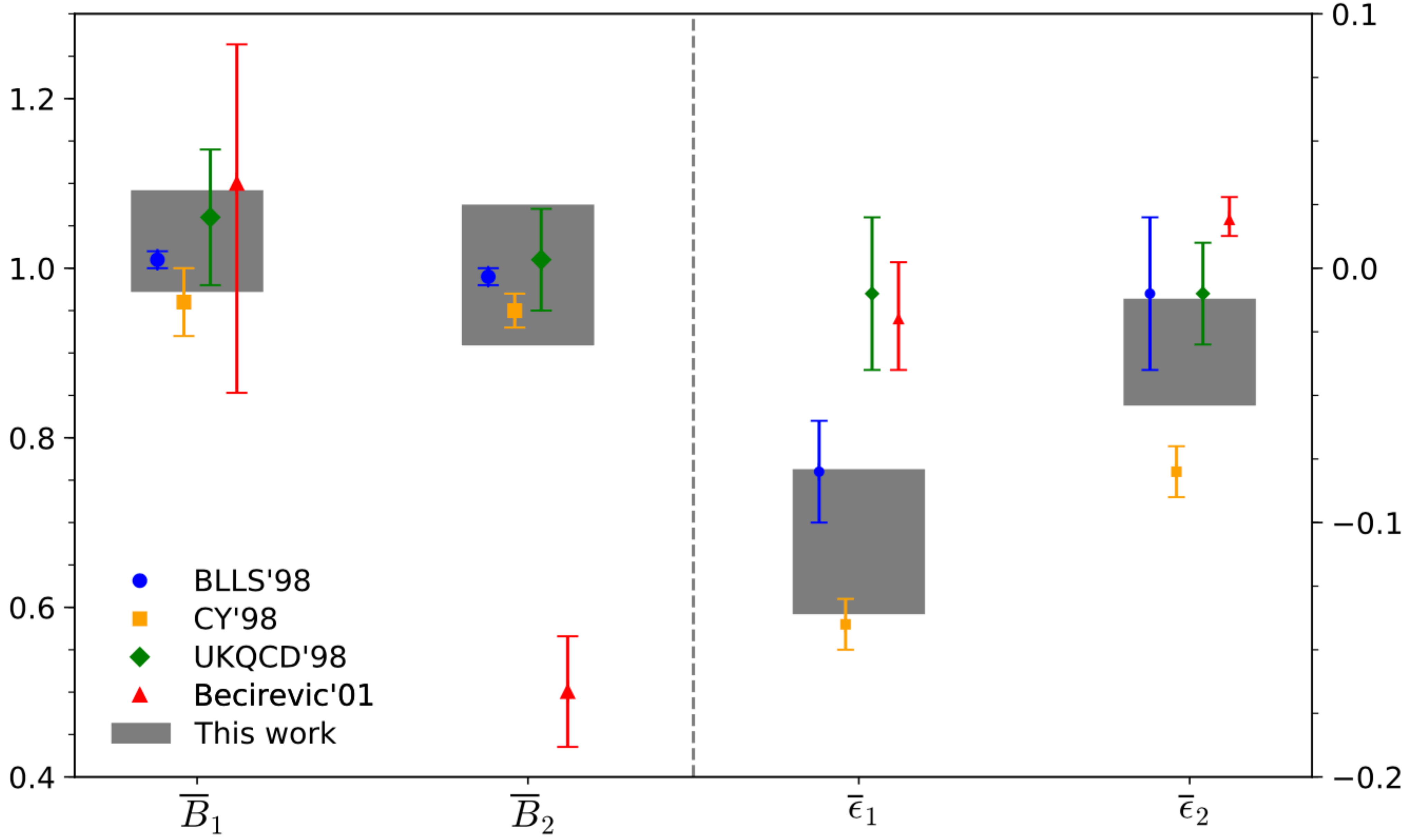
1806.00253 Grozin, Mannel, Pivovarov

Current status:

- **B1: SR might agree or might be lower than large FNAL/MILC**
- **B4/B4: SR seem to prefer FNAL/MILC**

Independent lattice determination urgently needed as well as higher precision in SR AND SR determination of Bs mesons

NEW RESULTS 2: B LIFETIMES



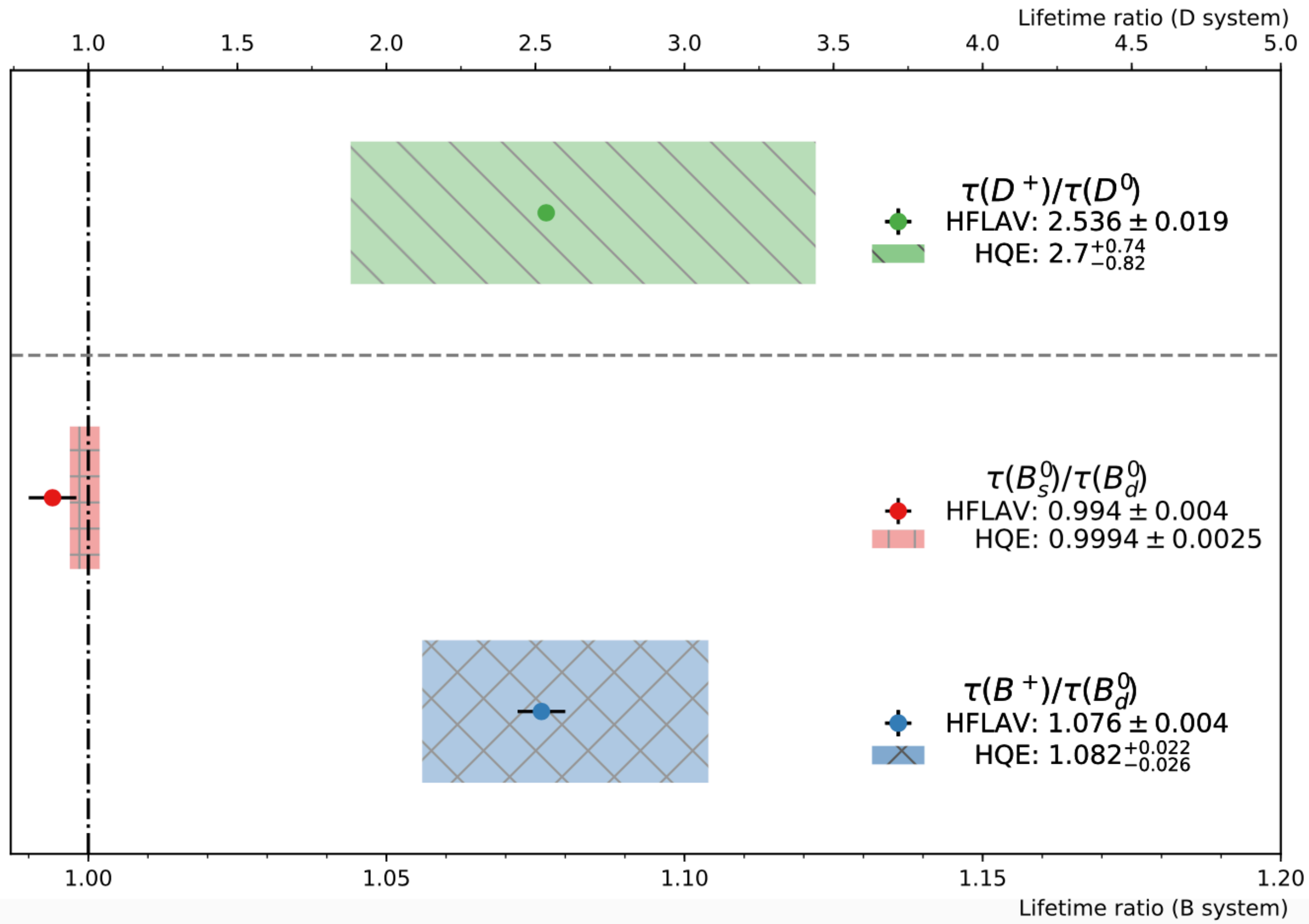
Kirk, Lenz, Rauh 1711.02100

- **Only modern determination - else: 2001**
- **Independent confirmation from lattice urgently needed!!!**

FINAL RESULTS: LIFETIMES

Uncertainty can be considerably reduced: NNLO Matching + dim 7

Question of HQE convergence is not a question of believes, but can be systematically studied!



Kirk, Lenz, Rauh

1711.02100

- HQE works even for D lifetimes! (roughly 30% precision)

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.7 = 1 + 16\pi^2 (0.25)^3 (1 - 0.34)$$

- B+ and Bs lifetime ratios agree perfectly with experiment
- Confirmation from lattice urgently needed
- Study more charmed hadrons

HEAVY QUARK EXPANSION

B mixing - decay rate differences

HOMEWORK

Diagonalise the matrix:

$$\begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix}$$

1) What are the eigenvalues in the case $\Gamma_{12} \ll M_{12}$ **?**

2) What are the eigenvalues in the general case?

Fundamentals of B-mixing 1

Mixing = Mass eigenstate differs from interaction eigenstate

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Quarks: CKM matrix
- Leptons: PMNS matrix
- Neutrinos: oscillations
- Electro-weak gauge bosons
- Neutral mesons

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad \begin{aligned} \nu_1(t) &= \nu_1(0)\cdot e^{im_1 t}, \\ \nu_2(t) &= \nu_2(0)\cdot e^{im_2 t}. \end{aligned}$$

$$\begin{pmatrix} W^+ \\ W^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} W^1 \\ W^2 \end{pmatrix},$$

$$\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\,\mu} \end{pmatrix},$$

1955 K^0-system → Rainer Wanke (Mainz): Kaon physics

1986 B_d-system

2006/12 B_s-system

→ Stephanie Hansmann-Menzemer (Heidelberg): Introduction to Flavour Physics (K^0, B_d, B_s)

2007/12 D^0-system → Eva Gersabeck (Manchester): Charm mixing & CPV

Fundamentals of B-mixing 2

Neutral mesons like B_d^0 and their anti particles $\bar{B}_d^0$ form a two state system, which can be described with a Schrödinger like equation

$$i\hbar\frac{\partial}{\partial t}\left(\begin{array}{c} B_d^0 \\ \bar{B}_d^0 \end{array}\right) = \hat{H}\left(\begin{array}{c} B_d^0 \\ \bar{B}_d^0 \end{array}\right) = \left(\begin{array}{cc} M_{11}^d - \frac{i}{2}\Gamma_{11}^d & 0 \\ 0 & M_{22}^d - \frac{i}{2}\Gamma_{22}^d \end{array}\right)\left(\begin{array}{c} B_d^0 \\ \bar{B}_d^0 \end{array}\right). \tag{265}$$

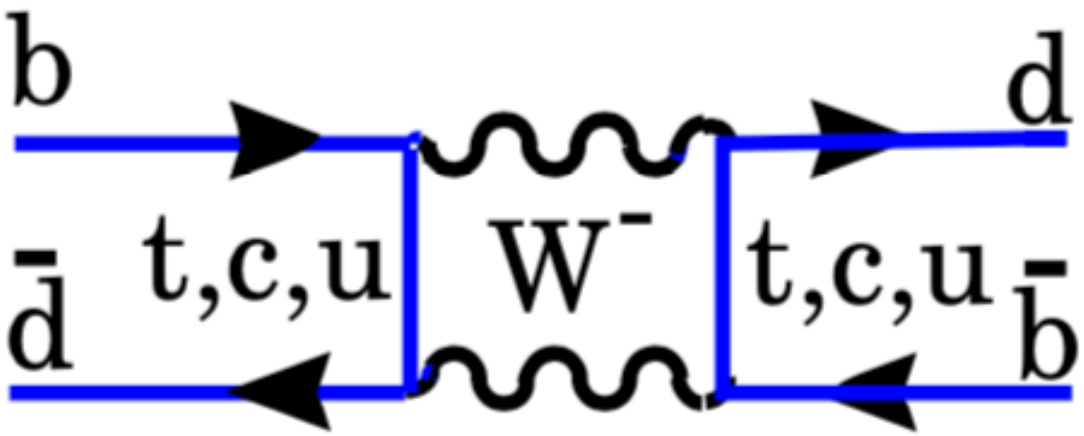
This is equivalent to the following time evolution of B mesons:

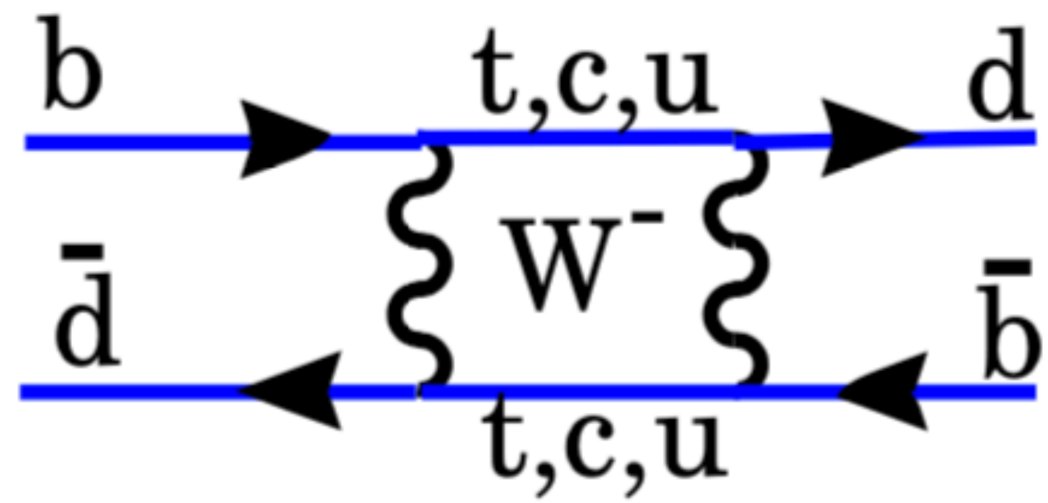
$$\Rightarrow B_i(t) = e^{\frac{1}{i\hbar}\left(M_{ii}^d - \frac{i}{2}\Gamma_{ii}^d\right)t} = e^{\frac{1}{i}\frac{1}{\hbar}M_{ii}^d t} e^{-\frac{1}{2}\frac{1}{\hbar}\Gamma_{ii}^d t}. \tag{266}$$

- $M_{11}^d(M_{22}^d)$ is the mass of the $B_d^0(\bar{B}_d^0)$-meson.
- $\Gamma_{11}^d(\Gamma_{22}^d)$ is the decay rate of the $B_d^0(\bar{B}_d^0)$-meson.
- CPT invariance implies $M_{11}^d = M_{22}^d$ and $\Gamma_{11}^d = \Gamma_{22}^d$.

Fundamentals of B-mixing 3

Due to the weak interaction, however, transitions of a B_d^0-meson to a $\bar{B}_d^0$ (and vise verca) are possible via the so-called **box diagrams**.





The box diagrams lead to off-diagonal terms in the Hamiltonian

$$\hat{H} = \begin{pmatrix} M_{11}^d - \frac{i}{2}\Gamma_{11}^d & M_{12}^d - \frac{i}{2}\Gamma_{12}^d \\ M_{21}^d - \frac{i}{2}\Gamma_{21}^d & M_{22}^d - \frac{i}{2}\Gamma_{22}^d \end{pmatrix} .$$

off-shell, like top, W
but also up and charm

on-shell, like up, charm

Fundamentals of B-mixing 4

Diagonalise the 2x2 matrix

$$\begin{aligned} B_{d,H} &= pB_d^0 - q\bar{B}_d^0 \,, \\ B_{d,L} &= pB_d^0 + q\bar{B}_d^0 \,, \end{aligned}$$

Mass eigenstates

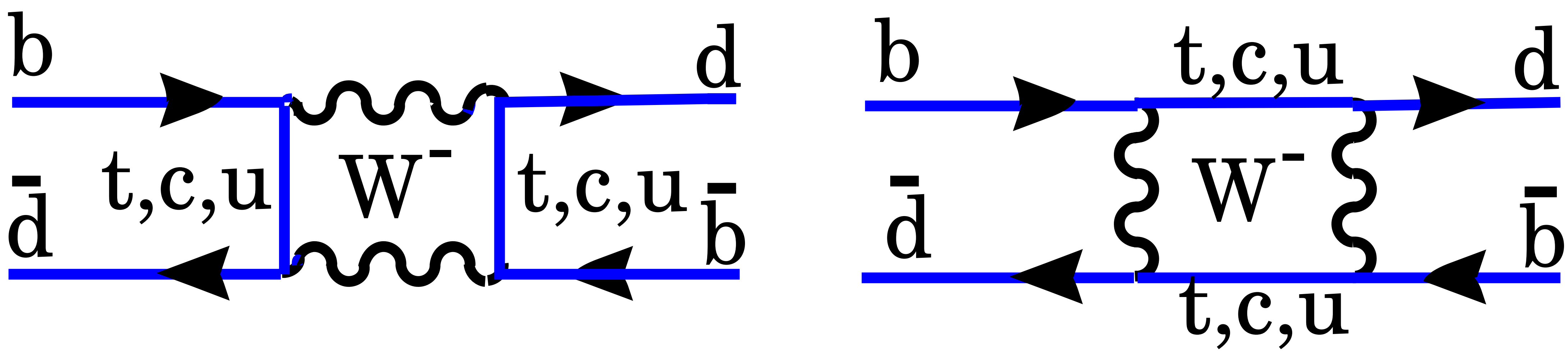
Interaction eigenstates

$$\begin{aligned} (\Delta M_d)^2 - \frac{1}{4}(\Delta\Gamma_d)^2 &= 4\left|M_{12}^d\right|^2 - \left|\Gamma_{12}^d\right|^2 \,, \\ \Delta M_d \cdot \Delta\Gamma_d &= -4\mathrm{Re}\left(M_{12}^d \Gamma_{12}^{d*}\right) \,, \\ \frac{q}{p} &= -\frac{\Delta M_d + \frac{i}{2}\Delta\Gamma_d}{2M_{12}^d - i\Gamma_{12}^d} \,. \end{aligned}$$

with

$$\begin{aligned} \Delta\Gamma_d &= \Gamma_L^d - \Gamma_H^d = \Delta\Gamma_d(M_{12}^d, \Gamma_{12}^d) \,, \\ \Delta M_d &= M_H^d - M_L^d = \Delta M_d(M_{12}^d, \Gamma_{12}^d) \,, \end{aligned}$$

Fundamentals of B-mixing 5



$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
  $|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi$ **(on-shell)**
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to X l \nu$ (*semi-leptonic*)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi$$

Fundamentals of B-mixing 6

Oscillations of B mesons

see e.g.

B physics at the Tevatron: Run II and beyond

K. Anikeev (MIT) *et al.*. Dec 2001. 583 pp.

e-Print: **hep-ph/0201071** | **PDF**

Mass eigenstates have obvious time evolution

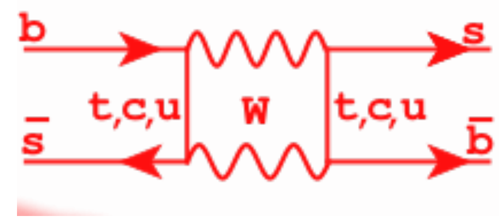
$$|B_{d,H/L}(t)\rangle \;=\; e^{-\left(iM^d_{H/L}+\Gamma^d_{H/L}/2\right)t}|B_{d,H/L}(0)\rangle\ .$$

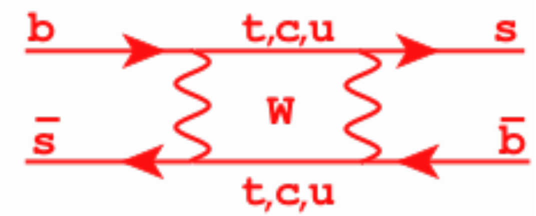
Transform to interaction eigenstates

$$|B^0_d(t)\rangle \;=\; g_+(t)|B^0_d\rangle + \frac{q}{p}g_-(t)|\bar{B}^0_d\rangle\ ,$$

$$|\bar{B}^0_d(t)\rangle \;=\; \frac{p}{q}g_-(t)|B^0_d\rangle + g_+(t)|\bar{B}^0_d\rangle\ ,$$

Fundamentals of B-mixing 7





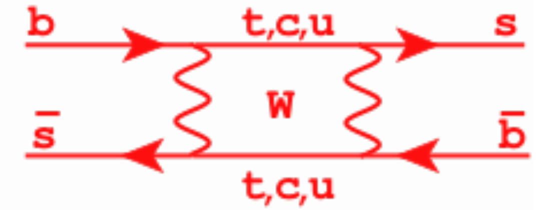
First we look at all the diagrams contributing to M_{12}^d. For each topology we get nine contributions

$$
\begin{aligned}
M_{12}^d \quad = \quad & \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t) + \\
& \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t) + \\
& \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t) \qquad (295)
\end{aligned}
$$

with the CKM structures $\lambda_q = V_{qd}^* V_{qb}$.
Next we can use unitarity of the CKM matrix ($\lambda_u + \lambda_c + \lambda_t = 0$) to eliminate λ_c.

$$
\begin{aligned}
M_{12}^d \quad = \quad & \lambda_u^2 \left(F(c,c) - 2F(u,c) + F(u,u) \right) \\
& + 2\lambda_u \lambda_t \left(F(u,t) - F(c,t) - F(u,c) + F(c,c) \right) \\
& + \lambda_t^2 \left(F(t,t) - 2F(c,t) + F(c,c) \right) \qquad (296)
\end{aligned}
$$

Fundamentals of B-mixing 8



- CKM elements

$$\begin{aligned} \lambda_u^2 &\propto \lambda^{6(7.512}\,, \\ \lambda_u\lambda_t &\propto \lambda^{6(6.756)}\,, \\ \lambda_t^2 &\propto \lambda^{6}\,, \end{aligned}$$

- Loop functions

$$F(p,q) \;=\; f_0 + f(m_q, m_p)$$

1. Constant term f_0 cancel
2. If internal quarks have equal masses, then M_12 vanishes

GIM cancellation

3. f depends strongly on (m_q/M_W)^2

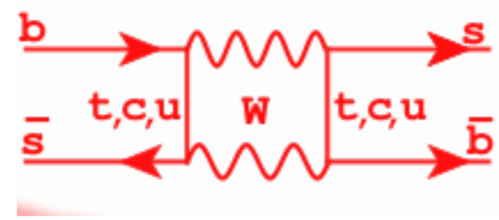
0.0000... 0.0003 4.5

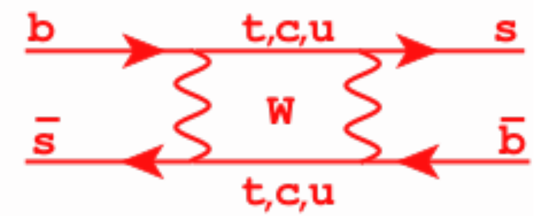
$$\frac{m_u^2}{M_W^2} \approx 0 \approx \frac{m_c^2}{M_W^2} \ll \frac{m_t^2}{M_W^2}$$

0.0000... 0.0000... 0.003

Charm system: $\frac{m_d^2}{M_W^2}, \frac{m_s^2}{M_W^2}, \frac{m_b^2}{M_W^2}$ **are all small !**

Fundamentals of B-mixing 8





$$\begin{aligned} M_{12}^d &= \lambda_u^2 \left(F(c,c) - 2F(u,c) + F(u,u)\right) \\ &+2\lambda_u\lambda_t \left(F(u,t) - F(c,t) - F(u,c) + F(c,c)\right) \\ &+\lambda_t^2 \left(F(t,t) - 2F(c,t) + F(c,c)\right) \end{aligned}$$

GIM

$$\propto \lambda_t^2 S(m_t^2/M_W^2) \,;$$

Inami-Lim function

$$S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2} \,.$$

In the Bs system an additional CKM suppression

$$\begin{aligned} \lambda_u^2 &\propto \lambda^{8(9.512)} \,, \\ \lambda_u\lambda_t &\propto \lambda^{6(6.756)} \,, \\ \lambda_t^2 &\propto \lambda^4 \,, \end{aligned}$$

In the charm system the internal b-quark is CKM suppressed, compared to down and strange

Fundamentals of B-mixing 9

CKM elements

2 loop QCD corrections

$$M_{12}^{s} = \frac{G_F^2}{12\pi^2}\lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s^0} \hat{\eta}_B ,$$

Weak coupling, W propagator

Inami-Lim function

non-perturbative matrix elements

via lattice or sum rules

$$\langle Q \rangle \equiv \langle \bar{B}_s^0 | Q | B_s^0 \rangle = \tfrac{8}{3} M_{B_s^0}^2 f_{B_s}^2 B(\mu).$$

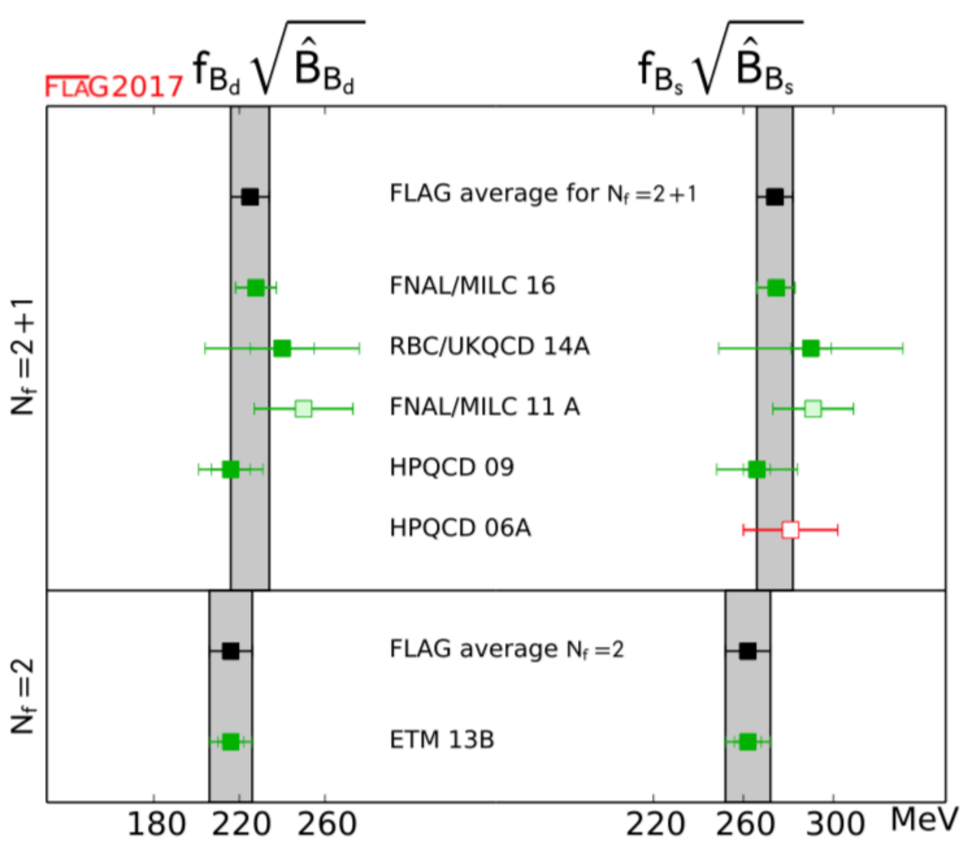
Franchesco Knechtli (Wuppertal): Lattice QCD

$$Q = \bar{s}^{\alpha}\gamma_{\mu}(1-\gamma_5)b^{\alpha} \times \bar{s}^{\beta}\gamma^{\mu}(1-\gamma_5)b^{\beta}.$$

Fundamentals of B-mixing 10

$$\Delta M_s^{\text{Exp}} = (17.757 \pm 0.021)\ \text{ps}^{-1}$$

SM error dominated by non-perturbative parameter



Source	$f_{B_s}\sqrt{\hat{B}}$	ΔM_s^{SM}
HPQCD14 [128]	(247 ± 12) MeV	$(16.2 \pm 1.7)\ \text{ps}^{-1}$
ETMC13 [129]	(262 ± 10) MeV	$(18.3 \pm 1.5)\ \text{ps}^{-1}$
HPQCD09 [130] = FLAG13 [131]	(266 ± 18) MeV	$(18.9 \pm 2.6)\ \text{ps}^{-1}$
FLAG17 [69]	$\mathbf{(274 \pm 8)}$ **MeV**	$\mathbf{(20.01 \pm 1.25)\ ps^{-1}}$
Fermilab16 [71]	(274.6 ± 8.8) MeV	$(20.1 \pm 1.5)\ \text{ps}^{-1}$
HQET-SR [76, 132]	(278^{+28}_{-24}) MeV	$(20.6^{+4.4}_{-3.4})\ \text{ps}^{-1}$
HPQCD06 [133]	(281 ± 20) MeV	$(21.0 \pm 3.0)\ \text{ps}^{-1}$
RBC/UKQCD14 [134]	(290 ± 20) MeV	$(22.4 \pm 3.4)\ \text{ps}^{-1}$
Fermilab11 [135]	(291 ± 18) MeV	$(22.6 \pm 2.8)\ \text{ps}^{-1}$

HEAVY QUARK EXPANSION

Total decay rate can be expanded in inverse powers of mb

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \dots$$

Each term in the series can be further expanded in the strong coupling

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \dots$$

Each term is a product of a perturbative function and the matrix element of **Delta B = 0 operators** **(lattice , sum rules)**

Mixing obeys a similar HQE

$$\Gamma_{12}^q = \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4 \Gamma_4 + \dots$$

Now **Delta B = 2 operators appear** **(lattice , sum rules)**

STATUS BEFORE 2017

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$< \|\mathrm{dim}\,6\| >$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$< \|\mathrm{dim}\,7\| >$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

THEORY UNCERTAINTIES IN MIXING

$\Delta\Gamma_s^{SM}$	This work
Central value	$0.088\ ps^{-1}$
$\delta(B_{\tilde{R}_2})$	14.8%
$\delta(f_{B_s}\sqrt{B})$	13.9%
$\delta(\mu)$	8.4%
$\delta(V_{cb})$	4.9%
$\delta(\tilde{B}_S)$	2.1%
$\delta(B_{R_0})$	2.1%
$\delta(\bar{z})$	1.1%
$\delta(m_b)$	0.8%
$\delta(B_{\tilde{R}_1})$	0.7%
$\delta(B_{\tilde{R}_3})$	0.6%
$\delta(B_{R_1})$	0.5%
$\delta(B_{R_3})$	0.2%
$\delta(m_s)$	0.1%
$\delta(\gamma)$	0.1%
$\delta(\alpha_s)$	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%
$\delta(\bar{m}_t(\bar{m}_t))$	0.0%
$\sum\delta$	22.8%

3 dominant uncertainties:

★ $$\langle R_2\rangle = -\frac{2}{3}\left[\frac{M_{B_s}^2}{m_b^{\mathrm{pow}2}} - 1\right] M_{B_s}^2 f_{B_s}^2 B_{R_2};$$ $$R_2 = \frac{1}{m_b^2}\,\bar{s}_\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho b_\alpha\, \bar{s}_\beta \gamma_\mu (1-\gamma_5) b_\beta$$

Dim 7 has never been done

★ $$\langle Q\rangle \equiv \langle \bar{B}_s^0|Q|B_s^0\rangle = \tfrac{8}{3} M_{B_s^0}^2 f_{B_s}^2 B(\mu)$$ $$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta$$

Dim 6 is done on the lattice
newest results (**Fermilab MILC 1602:03560**)
indicate a tension with experiment

★ **NNLO QCD has not been done**

CP violation in the Bs system
Marina Artuso, Guennadi Borissov, Alexander Lenz
Rev.Mod.Phys. 88 (2016) no.4,045002

NEWS

	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$<\|\mathrm{dim}\,6\|>$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$<\|\mathrm{dim}\,7\|>$
B+	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
Bs	1985 -1996 ✓	2002 ✓	✗	2001 ✗	2003 ✓	✗	✗
G12s	1985 -1996 ✓	1998 -2006 ✓	✗	-2016 ✗	1996 ✓	✗	✗
G12d	1985 -1996 ✓	2003 -2006 ✓	✗	-2016 ✗	2003 ✓	✗	✗

First steps: Asatrian et al 1709.02160

Sum rules: Kirk, Lenz, Rauh 1711.02100

HPQCD in progress, see LATTICE 2016, 2017

Sum rules: Kirk, Lenz, Rauh in progress

HQE FOR B-MIXING

Fit results from ATLAS, CDF, CMS, D0 and LHCb data	without constraint from effective lifetime measurements	with constraints I and II	**with constraints I, II and III**
Γ_s	0.6640 ± 0.0020 ps^{-1}	0.6627 ± 0.0020 ps^{-1}	**0.6625 ± 0.0018 ps^{-1}**
$1/\Gamma_s$	1.506 ± 0.005 ps	1.509 ± 0.004 ps	**1.509 ± 0.004 ps**
$\tau_{Short} = 1/\Gamma_L$	1.415 ± 0.007 ps	1.414 ± 0.006 ps	**1.414 ± 0.006 ps**
$\tau_{Long} = 1/\Gamma_H$	1.609 ± 0.010 ps	1.618 ± 0.010 ps	**1.619 ± 0.009 ps**
$\Delta\Gamma_s$	$+0.085 \pm 0.006$ ps^{-1}	$+0.089 \pm 0.006$ ps^{-1}	**$+0.090 \pm 0.005$ ps^{-1}**
$\Delta\Gamma_s/\Gamma_s$	$+0.128 \pm 0.009$	$+0.135 \pm 0.008$	**$+0.135 \pm 0.008$**
correlation $\rho(\Gamma_s, \Delta\Gamma_s)$	-0.193	-0.153	**-0.082**

HFLAV 2018

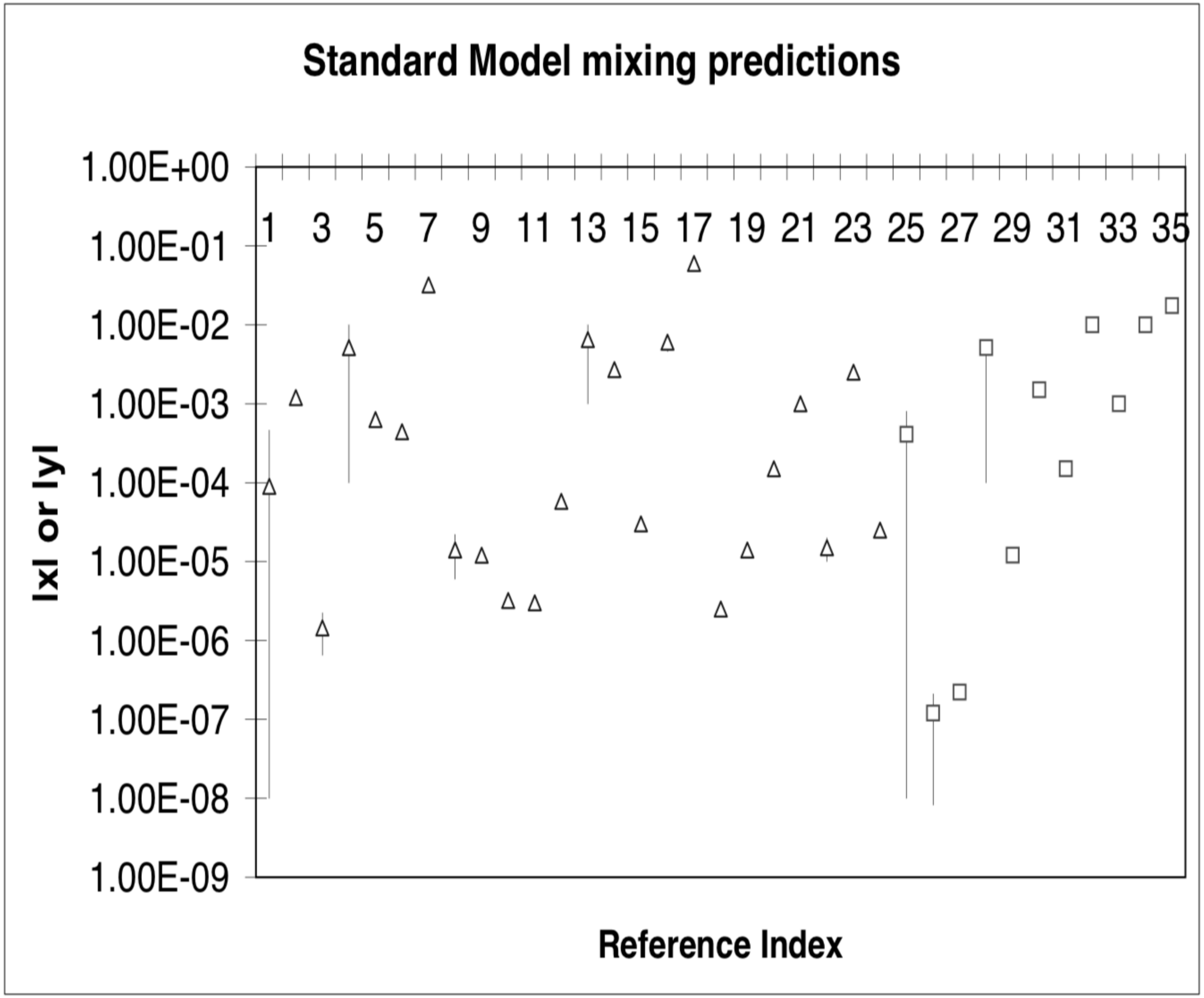
Again: perfect agreement of experiment and theory

Observable	SM - conservative	SM - aggressive
ΔM_s	$(18.3 \pm 2.7)\ \text{ps}^{-1}$	$(20.31 \pm 1.34)\ \text{ps}^{-1}$
$\Delta\Gamma_s$	$(0.088 \pm 0.20)\ \text{ps}^{-1}$	$(0.1001 \pm 0.0134)\ \text{ps}^{-1}$
a_{sl}^s	$(2.22 \pm 0.27) \cdot 10^{-5}$	$(2.272 \pm 0.249) \cdot 10^{-5}$
$\frac{\Delta\Gamma_s}{\Delta M_s}$	$48.1\,(1 \pm 0.173) \cdot 10^{-4}$	$49.3\,(1 \pm 0.117)$
ΔM_d	$(0.528 \pm 0.078)\ \text{ps}^{-1}$	$(0.615 \pm 0.053)\ \text{ps}^{-1}$
$\Delta\Gamma_d$	$(2.61 \pm 0.59) \cdot 10^{-3}\ \text{ps}^{-1}$	$(3.10 \pm 0.49) \cdot 10^{-3}\ \text{ps}^{-1}$
a_{sl}^d	$(-4.7 \pm 0.6) \cdot 10^{-4}$	$(-4.88 \pm 0.54) \cdot 10^{-4}$
$\frac{\Delta\Gamma_d}{\Delta M_d}$	$49.4\,(1 \pm 0.172) \cdot 10^{-4}$	$50.4\,(1 \pm 0.130)$

Jubb, Kirk, AL, Tetlalmatzi-Xolocotzi 2017

D MIXING

Charm theory is notoriously difficult



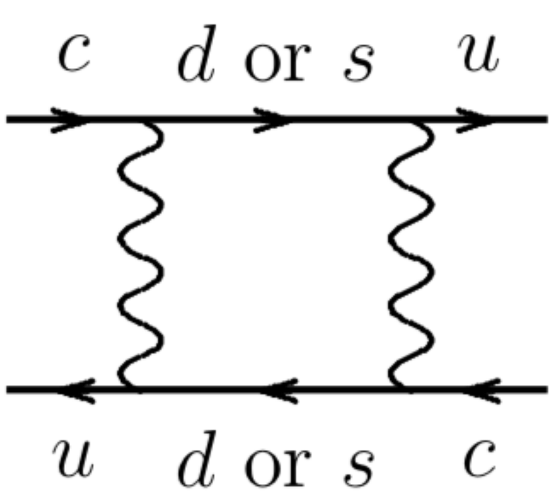
modified Nelson plot from A. Petrov hep-ph/0311371

Charm mixing - Theory 1

Flavour Eigenstates

$$|D^0\rangle = |c\bar{u}\rangle \qquad |\bar{D}^0\rangle = |\bar{c}u\rangle$$

Mixing due to box diagrams



Mass Eigenstates

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

Charm mixing - Theory 2

Diagonalise mass and decay rate matrix

$$\Delta M_D^2 - \frac{1}{4}\Delta\Gamma_D^2 = 4\left|M_{12}^D\right|^2 - \left|\Gamma_{12}^D\right|^2 ,$$

$$\Delta M_D \Delta\Gamma_D = 4\left|M_{12}^D\right|\left|\Gamma_{12}^D\right|\cos(\phi_{12}^D) ,$$

mass difference $\Delta M_D = M_1 - M_2$

decorate difference $\Delta\Gamma_D = \Gamma_2 - \Gamma_1$

absorptive part of box diagram (on-shell) Γ_{12}^D

dispersive part of box diagram (off-shell) M_{12}^D

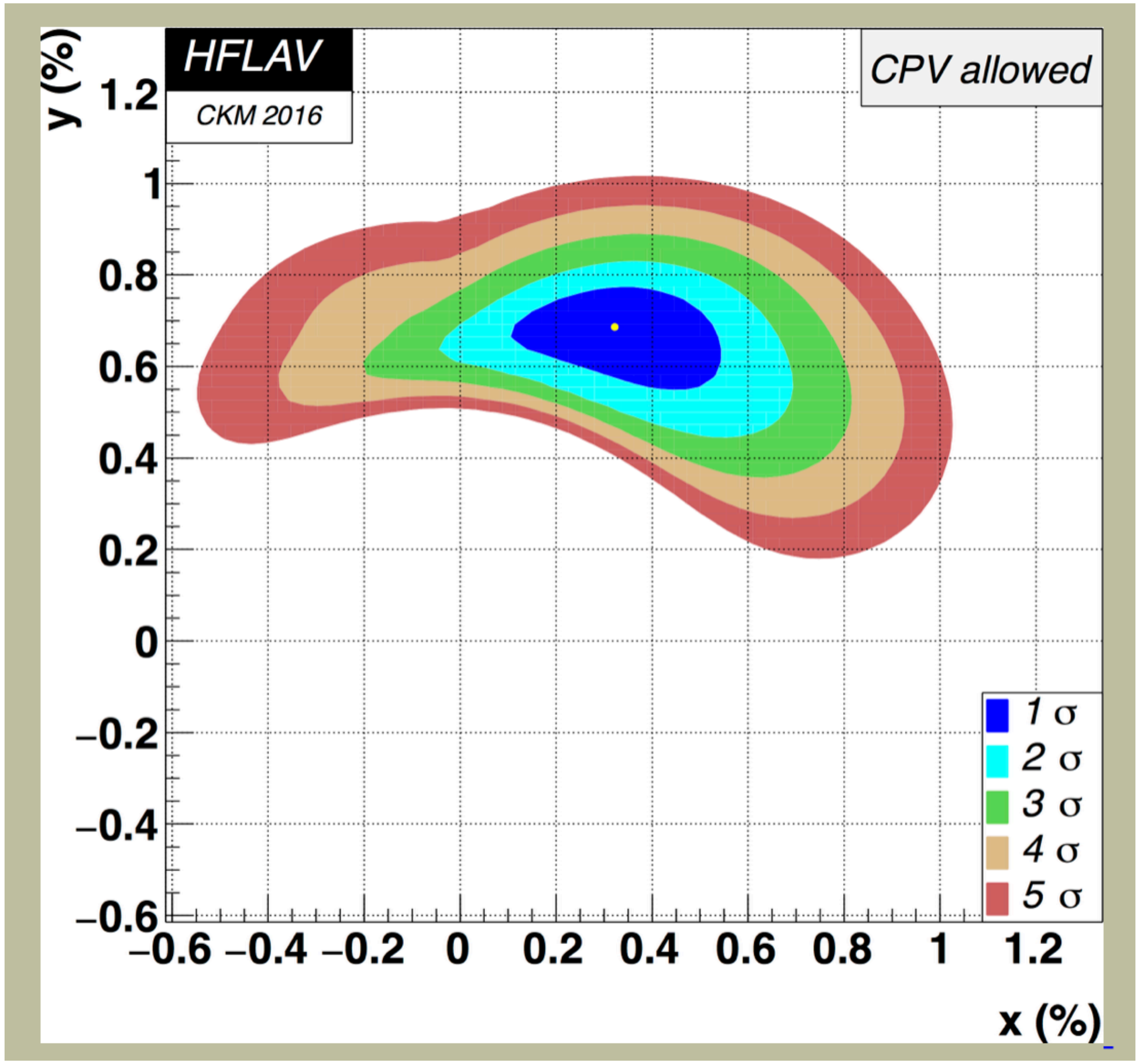
relative phase $\phi_{12}^D = -\arg(-M_{12}^D/\Gamma_{12}^D)$

Charm mixing - Experiment

Experimental situation

$$x := \frac{\Delta M_D}{\Gamma_D} = 0.32\% \pm 0.14\% \,,$$
$$y := \frac{\Delta \Gamma_D}{2\Gamma_D} = 0.61\% \pm 0.07\% \,,$$

HFLAV 2016/2018



- Small values
- non-vanishing x not yet confirmed

Charm mixing - Theory 3

Crucial differences compared to B mixing

1) No simple formulae like $\Delta M_{B_S} = 2|M_{12}^{B_s}|$

both Γ_{12}^D and M_{12}^D have to be known!

but there is a bound $\Delta\Gamma_D \leq 2|\Gamma_{12}^D|$

Nierste 0904.1869
Jubb et al. 1603.07770

2) GIM cancellation vs CKM hierarchy: $\lambda_b \ll \lambda_s$, but complex!!!

$$\Gamma_{12}^D = \qquad -\lambda_s^2 \left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd}^D - \Gamma_{dd}^D\right) - \lambda_b^2 \Gamma_{dd}^D \,,$$

$$M_{12}^D = \lambda_s^2 \left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D\right] + 2\lambda_s\lambda_b \left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D\right] + \lambda_b^2 \left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D\right] \,.$$

Charm mixing - Theory 3

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survives in SU(3)F limit!

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$$M_{12}^D = \lambda_s^2 \left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D\right] + 2\lambda_s\lambda_b \left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D\right] + \lambda_b^2 \left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D\right] \,.$$

Charm mixing - Theory 3

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but there is a bound $\Delta\Gamma_D \leq 2|\Gamma_{12}^D|$

2) GIM cancellation vs CKM hierarchy: $\lambda_b \ll \lambda_s$, but complex!!!

CPV

survives in SU(3)F limit!

dominant for B mixing

$$\Gamma_{12}^D = -\lambda_s^2\left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D\right) + 2\lambda_s\lambda_b\left(\Gamma_{sd}^D - \Gamma_{dd}^D\right) - \lambda_b^2\Gamma_{dd}^D\,,$$

$$M_{12}^D = \lambda_s^2\left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D\right] + 2\lambda_s\lambda_b\left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D\right] + \lambda_b^2\left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D\right]\,.$$

Charm mixing - Theory 4

Two theory approaches for calculating D mixing

1) **Inclusive** approach

calculate on **quark level**

Georgi 9209291
Ohl, Ricciardi,Simmons 9301212
Bigi, Uraltsev 0005089
Bobrowski et al 1002.4794

2) **Exclusive** approach

calculate on **hadron level**

Falk et al 0110317
Falk et al 0402204
Cheng, Chiang 1005.1106
Jiang et al1705.07335

Due to extreme GIM cancellation very high precision necessary!!!

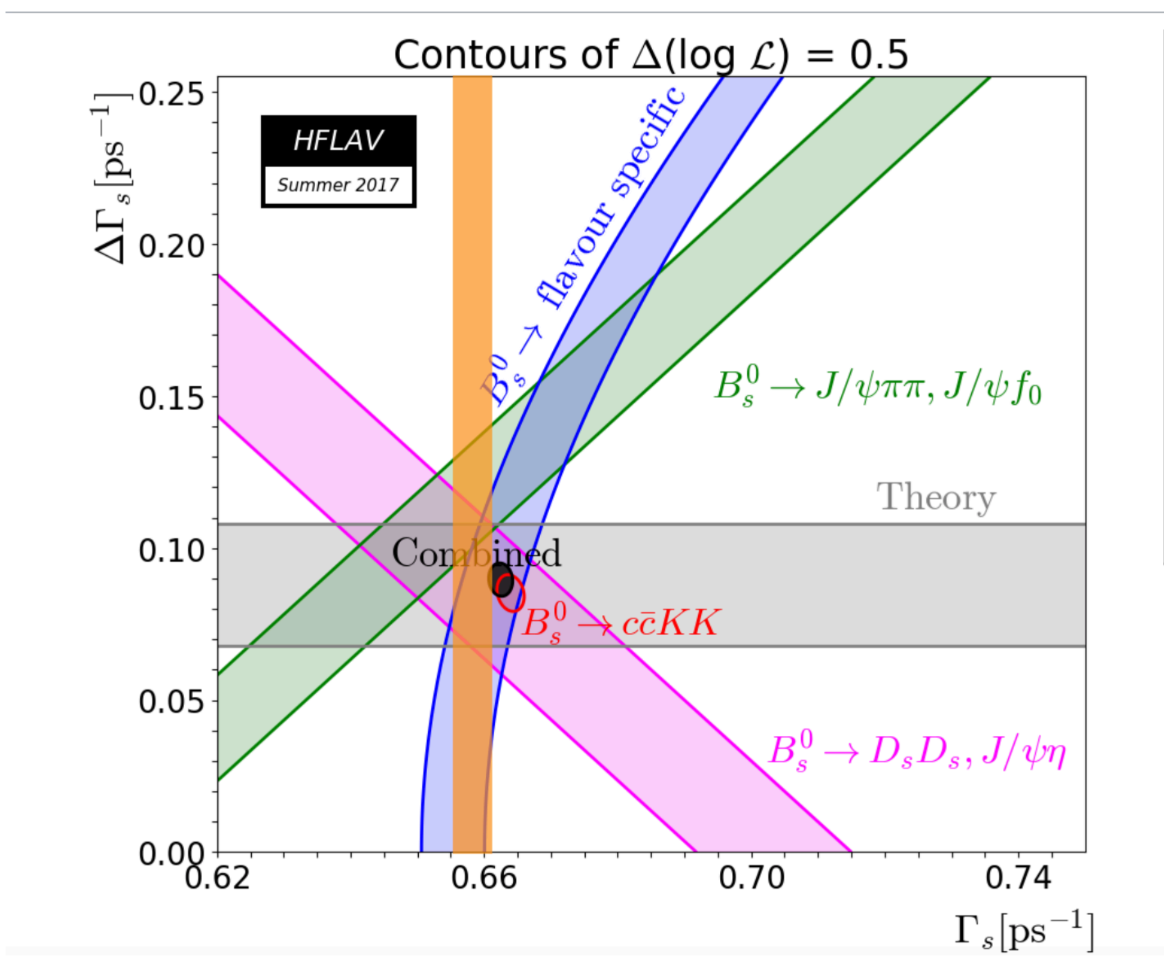
Charm mixing - Theory 5

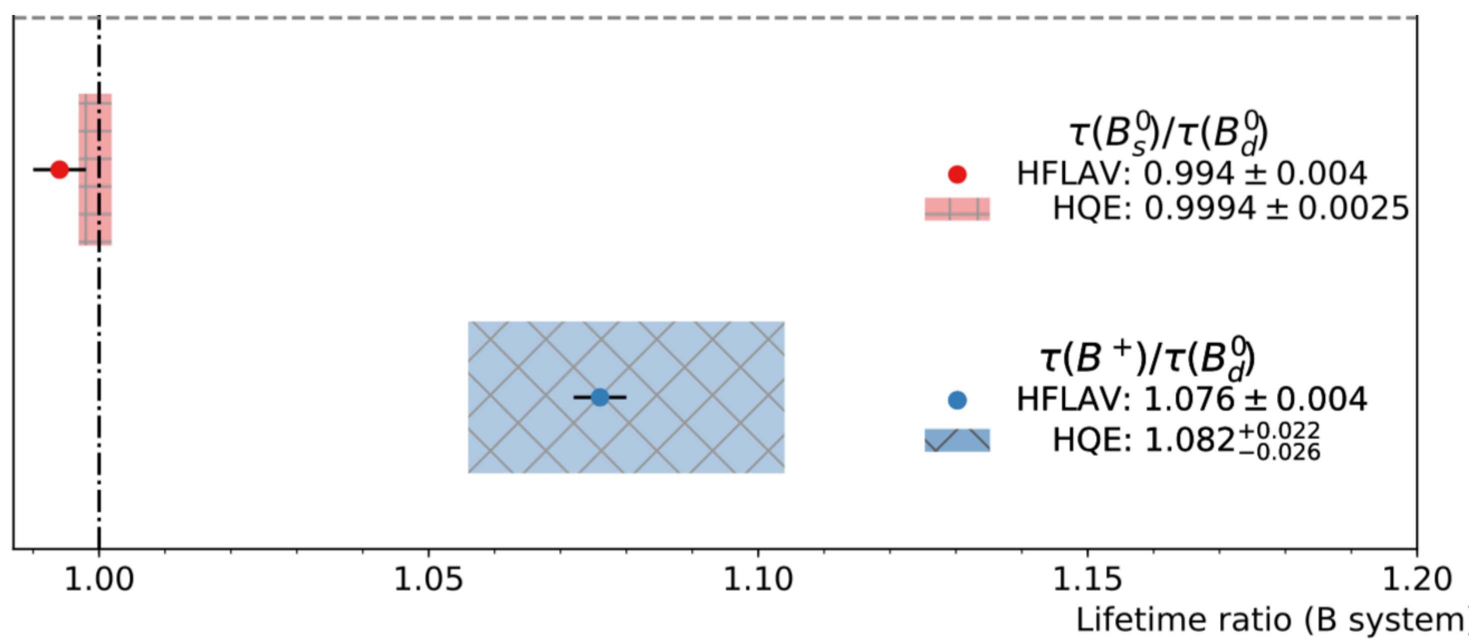
The Heavy Quark Expansion

Expansion in Λ / m_Q

Voloshin, Shifman 1983, 1985
Bigi, Uraltsev 1992
Bigi, Uraltsev, Vainshtein 1992
Blok, Shifman 1992

The HQE works well in the B-system



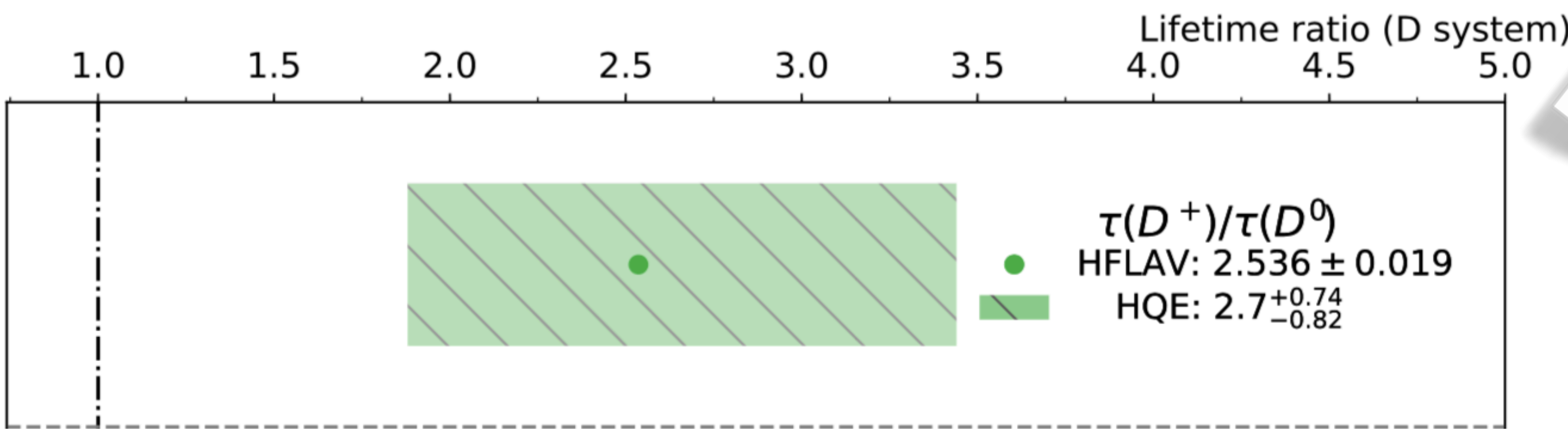


Kirk, AL, Rauh 1711.02100

Charm mixing - Theory 6

$\Lambda/m_c \approx 3\Lambda/m_b$ - could still give some reasonable estimates!

Look in systems without GIM cancellation: D-lifetimes



NEW 3-loop sum rules

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.7 = 1 + 16\pi^2 \, (0.25)^3 \, (1 - 0.34)$$

Kirk, AL, Rauh 1711.02100
pert. NLO-QCD:
AL, Rauh 1305.3588

Expansion parameter for HQE in charm = 0.3
not a back of envelope statement, but real calculations

d=6 calculated with sum rules
lattice confirmation urgently needed

d=7 estimated in vacuum insertion approximation
do sum rule/lattice

Charm mixing - Theory 7

The HQE is successful in the B system and for D meson lifetimes

=> apply it for D-mixing

Charm mixing - Theory 7

The HQE is successful in the B system and for D meson lifetimes

=> apply it for D-mixing

$$y_D^{\rm HQE} \approx \lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) \approx 1.7 \cdot 10^{-4} y_D^{\rm Exp.}$$

How can this be?

Charm mixing - Theory 7

The HQE is successful in the B system and for D meson lifetimes

=> apply it for D-mixing

$$y_D^{\text{HQE}} \approx \lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} - \Gamma_{12}^{dd} \right) \approx 1.7 \cdot 10^{-4} y_D^{\text{Exp.}}$$

How can this be?

Look only at a single diagram:

$$y_D^{\text{HQE}} \neq \lambda_s^2 \Gamma_{12}^{ss} \tau_D = 3.7 \cdot 10^{-2} \approx 5.6 y_D^{\text{Exp.}}$$

pert. calculation: Bobrowski et al 1002.4794
latice input: ETM 1403.7302; 1505.06639; FNAL/MILC 1706.04622

The problem seems to originate in the extreme GIM cancellations

Charm mixing - Theory 7

The HQE is successful in the B system and for D meson lifetimes

=> apply it for D-mixing

$$\Gamma_{12}^D = -\lambda_s^2 \left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd}^D - \Gamma_{dd}^D\right) - \lambda_b^2 \Gamma_{dd}^D \,,$$

$$\begin{aligned} 10^7 \Gamma_{12}^{D=6,7} = & -14.6409 + 0.0009i && (1^{\text{st}}\ \text{term}) \\ & -6.68 - 15.8i && (2^{\text{nd}}\ \text{term}) \\ & +0.27 - 0.28i && (3^{\text{rd}}\ \text{term}) \end{aligned}$$

Bobrowski et al 1002.4794

Important observation for CPV

Charm mixing - Theory 8

What could have gone wrong in D-mixing?

1. **Duality violations - break down of HQE**

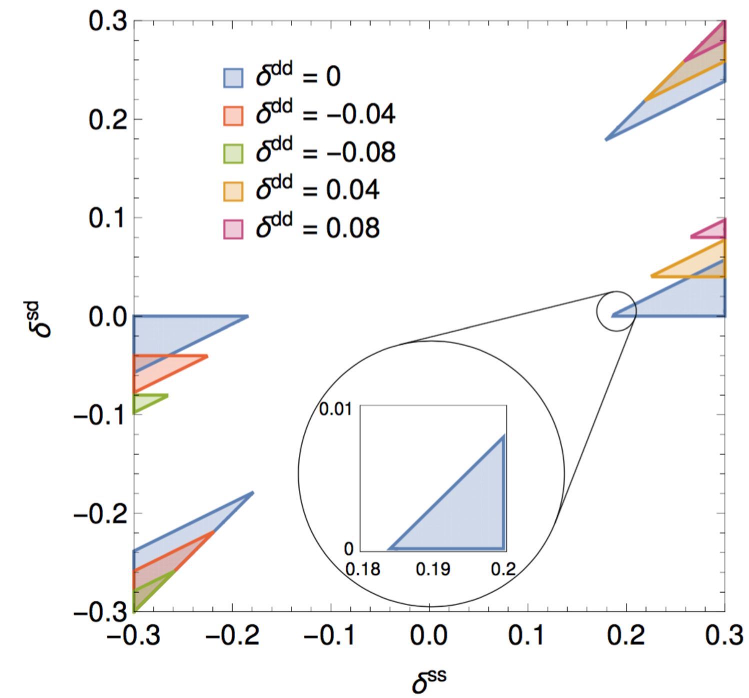
$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1+\delta^{ss}) \, ,$$

$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1+\delta^{sd}) \, ,$$

$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1+\delta^{dd}) \, ,$$

20% of duality violation is sufficient to explain experiment

Jubb, Kirk, AL, Tetlalmatzi-Xolocotzi 2016



2. **Higher dimensions** Georgi 9209291; Ohl, Ricciardi,Simmons 9301212; Bigi, Uraltsev 0005089

Idea: GIM cancellation is lifted by higher orders in the HQE - overcompensating the 1/mc suppression.

Partial calculation of D=9 yields an enhancement - but not to the experimental value Bobrowski, AL, Rauh 2012

3. **New Physics is present and we cannot proof it :-)**

Exclusive approach

$$\Gamma_{12}^D = \sum_n \rho_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle \,,$$

$$M_{12}^D = \sum_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=2} | D^0 \rangle + P \sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2} \,,$$

Cannot be calculated yet

Estimate phase space effects for y: Falk et al 0110317

- assume pert. SU(3)F breaking
- neglect 3 family
- neglect SU(3)F breaking in matrix elements

$y \approx 1\%$

Mass difference from a dispersion relation Falk et al 0402204 $x \approx y$

Exp. data Cheng, Chiang 1005.1106 $x \propto \mathcal{O}(0.1\%)$ $y \propto \mathcal{O}(few\ 0.1\%)$

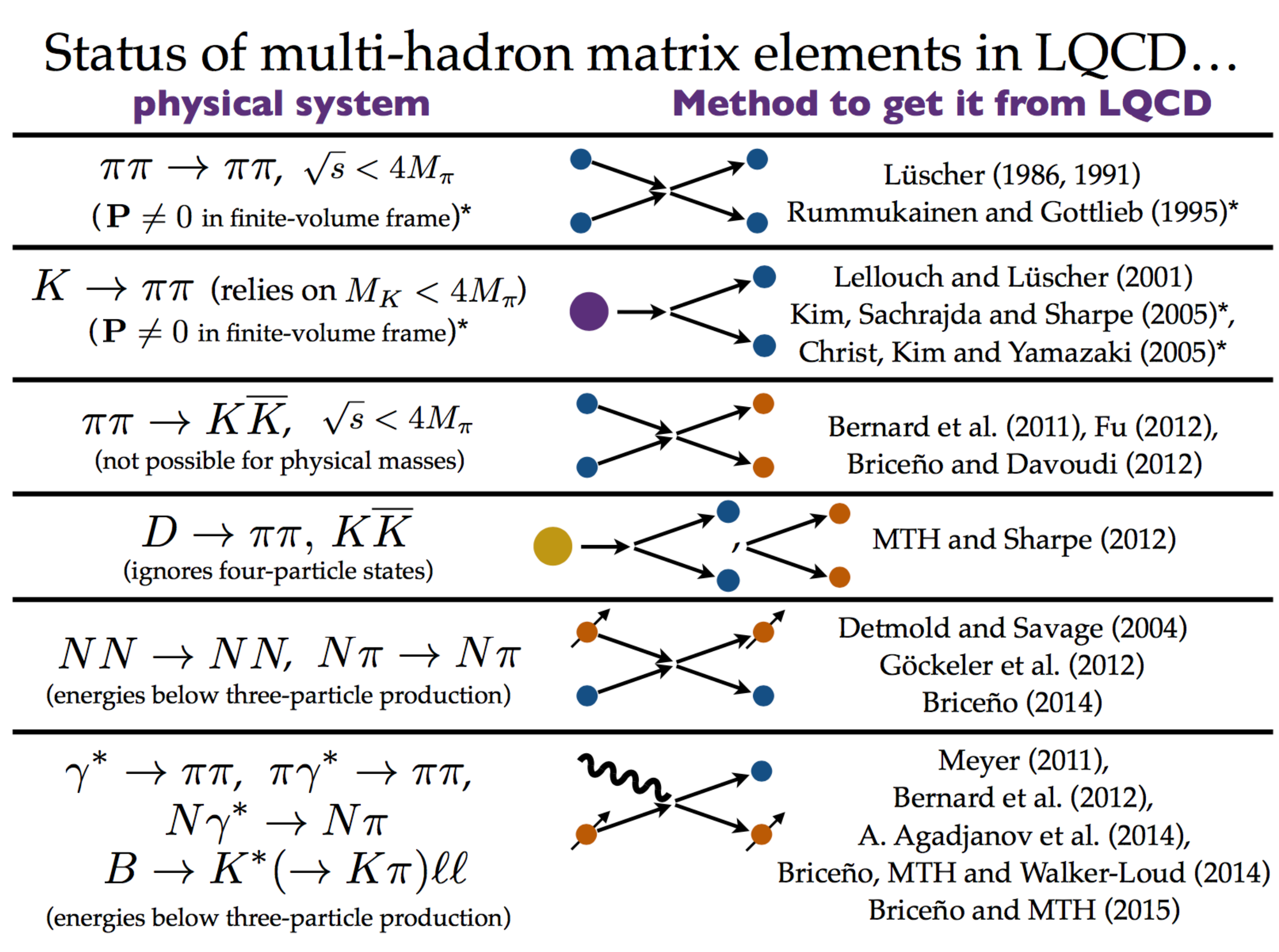
U-Spin sum rule Gronau, Rosner 2012

Factorisation-assisted topological amplitude approach
Jiang et al1705.07335 $y \approx 0.2\%$

Direct lattice determination

Still a very long way!
But not completely crazy anymore!

1. **Multiple-channel generalization of Lellouch-Luscher formula**
(170) Maxwell T. Hansen, Stephen R. Sharpe (Washington U., Seattle). Apr 2012. 15 pp.
Published in **Phys.Rev. D86 (2012) 016007**
DOI: 10.1103/PhysRevD.86.016007
e-Print: **arXiv:1204.0826** **[hep-lat]** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server
Detailed record - Cited by 170 records 100+


Status of multi-hadron matrix elements in LQCD…

physical system	Method to get it from LQCD
$\pi\pi \to \pi\pi,\ \sqrt{s} < 4M_\pi$ ($\mathbf{P} \neq 0$ in finite-volume frame)*	Lüscher (1986, 1991), Rummukainen and Gottlieb (1995)*
$K \to \pi\pi$ (relies on $M_K < 4M_\pi$) ($\mathbf{P} \neq 0$ in finite-volume frame)*	Lellouch and Lüscher (2001), Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*
$\pi\pi \to K\overline{K},\ \sqrt{s} < 4M_\pi$ (not possible for physical masses)	Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
$D \to \pi\pi,\ K\overline{K}$ (ignores four-particle states)	MTH and Sharpe (2012)
$NN \to NN,\ N\pi \to N\pi$ (energies below three-particle production)	Detmold and Savage (2004), Göckeler et al. (2012), Briceño (2014)
$\gamma^* \to \pi\pi,\ \pi\gamma^* \to \pi\pi,\ N\gamma^* \to N\pi,\ B \to K^*(\to K\pi)\ell\ell$ (energies below three-particle production)	Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014), Briceño and MTH (2015)

slide by Max Hansen

Theory to-do-list

Non-perturbative matrix elements for D-meson lifetimes

- **d= 6 lattice - check sum rules**
- **d=7 with sum rules, lattice**
- **NNLO matching a la** Grozin, Mannel, Pivovarov 1806.00253
- **charmed baryons**

Determine higher dimension contributions to Gamma_12

- **D=9**
- **D=12**

Determine M_12

Have a good idea for improving exclusive approaches

First direct lattice studies for D-mixing
(probably similar time scale as HL-LHC)

Charm Theory Task Force

- Joachim Brod, TU Dortmund
- Svjetlana Fajfer, Ljubljana
- Alexander Kagan, Cincinnati
- Alexander Lenz, IPPP Durham

October 2017, CERN
HL/HE Questions in charm

April 2018, Fermilab
Absorptive and dispersive CPV in D mixing (relation to experiment)

- Theory predictions