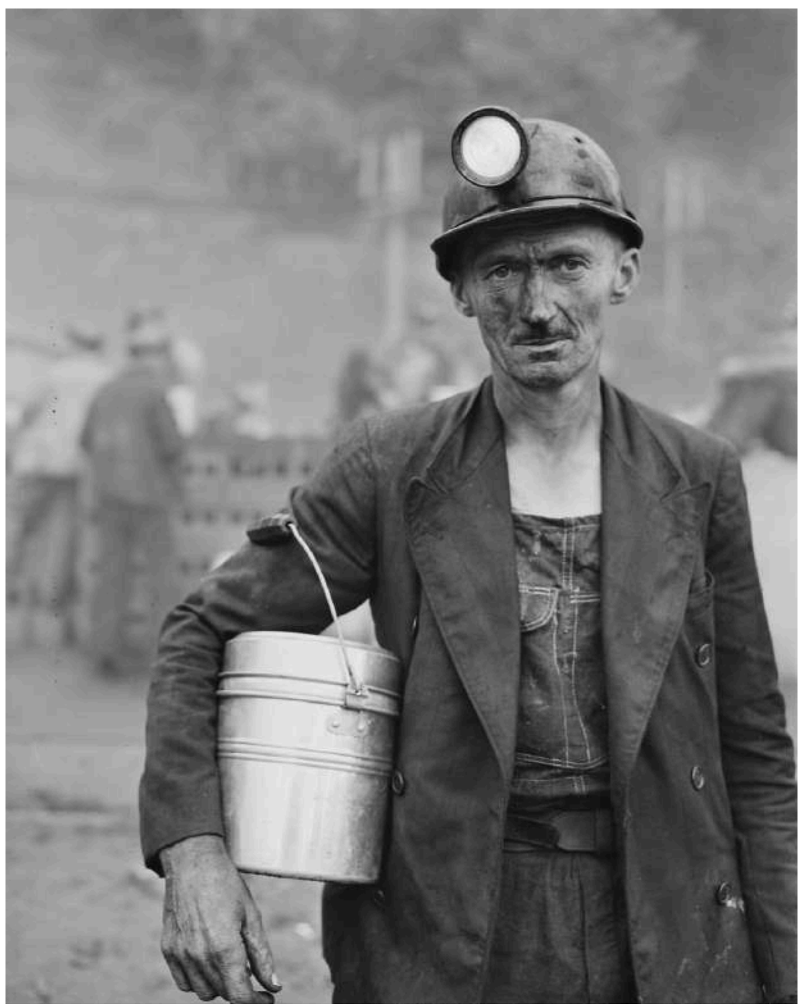
Quark Hadron Duality
vs.
Heavy Quark Expansion

A phenomenological point of view at a rather formal workshop



Alexander Lenz
IPPP Durham

Bridging Perturbative and Non-perturbative Physics
Primosten, 8.10.2019

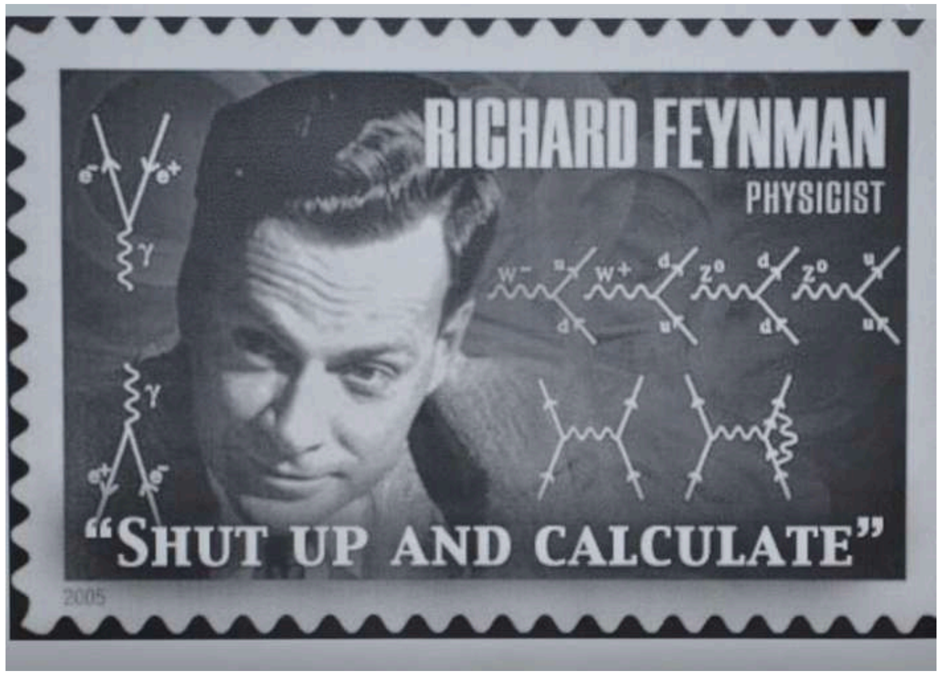
Messages from the machine room to the top deck

Z',

Taken from Thomas Mannel

Outline

- Quark Hadron Duality
- The Heavy Quark Expansion and what could go wrong
- Old problems: Lambda_b lifetime and friends
- Theoretical approach: try to solve QCD
- Pragmatical approach: Shut up and calculate
- Conclusions

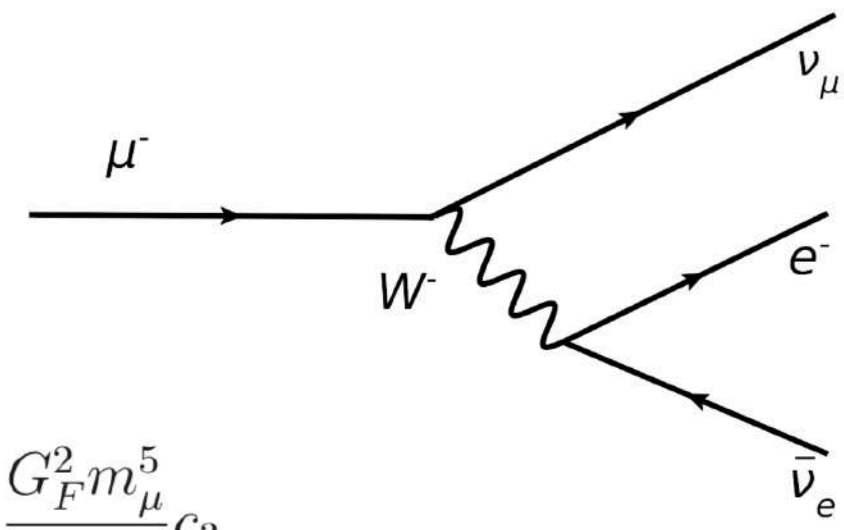


Probably due to Mermin and not Feynman https://physicstoday.scitation.org/doi/10.1063/1.1768652

Quark Hadron Duality

Experiment at Hadron Level - Calculation at Quark-Gluon level

Muon decay:
Simple and unambiguous



$$\Gamma_{\mu\to\nu_\mu+e+\bar{\nu}_e} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e}{m_\mu}\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} c_{3,\mu} \, .$$

$$c_{3,\mu} = f\left(\frac{m_e}{m_\mu}\right)\left[1 + \frac{\alpha}{4\pi} 2 \left(\frac{25}{4} - \pi^2\right)\right]$$

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln(x)$$

Gives quite a good description of nature
for higher accuracy
include
higher order corrections

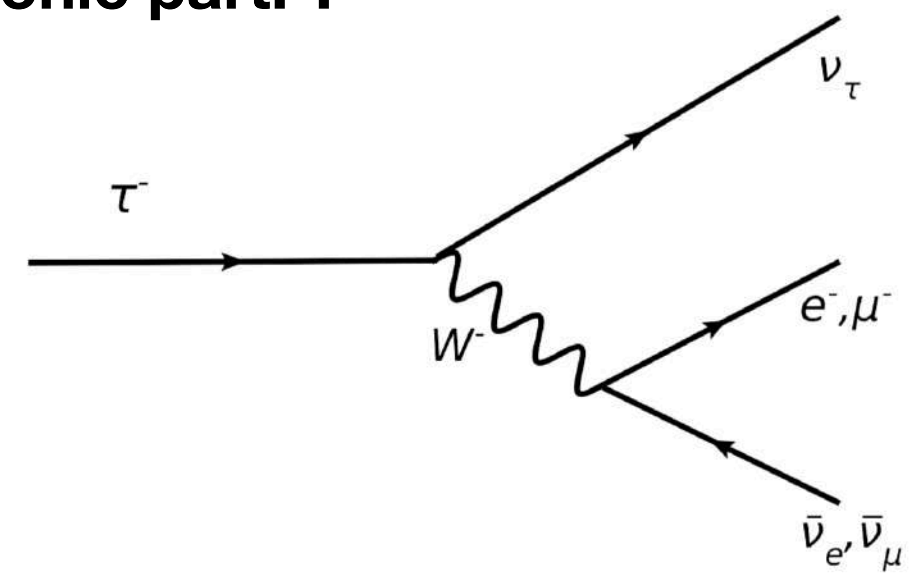
$$\tau_\mu^{Theo.} = 2.18776 \cdot 10^{-6}\,\mathrm{s}$$

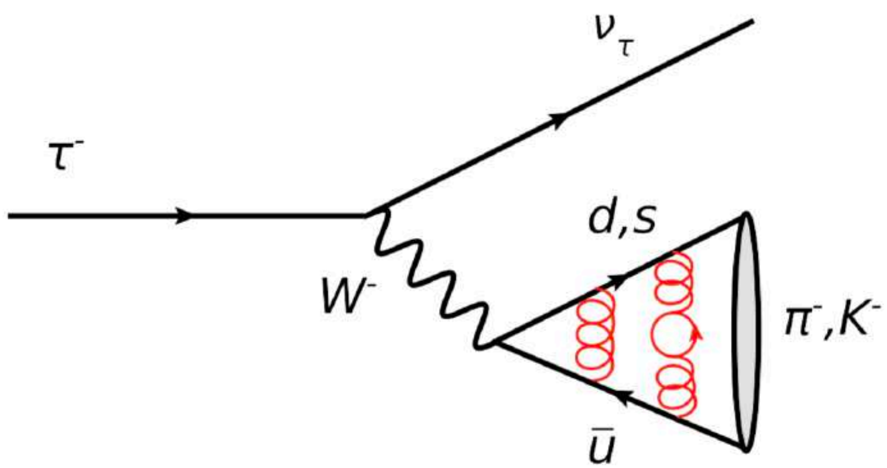
$$\tau_\mu^{Exp.} = 2.1969811(22) \cdot 10^{-6}\,\mathrm{s}$$

Quark Hadron Duality

Experiment at Hadron Level - Calculation at Quark-Gluon level

Tau-decay:
-Leptonic part as simple aus muon decays
-hadronic part: ?





naive, tree-level quark level calculation

$$\Gamma_\tau = \frac{G_F^2 m_\tau^5}{192\pi^3}\left[f\left(\frac{m_e}{m_\tau}\right) + f\left(\frac{m_\mu}{m_\tau}\right) + N_c\left|V_{ud}\right|^2 g\left(\frac{m_u}{m_\tau}, \frac{m_d}{m_\tau}\right) + N_c\left|V_{us}\right|^2 g\left(\frac{m_u}{m_\tau}, \frac{m_s}{m_\tau}\right)\right]$$

agrees quite well with experiment

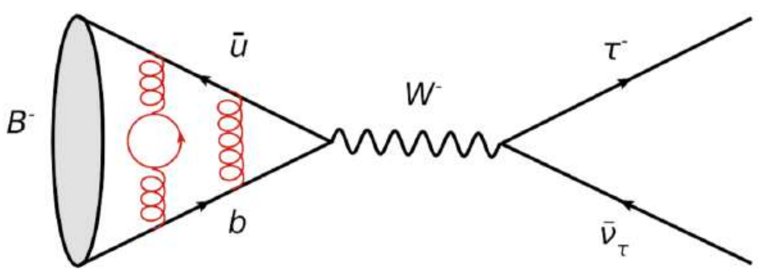
$\tau_\tau^{Exp.} = 2.906(1) \cdot 10^{-13}\ \mathrm{s}$ **vs.** $\tau_\tau^{Theo.} = 3.26707 \cdot 10^{-13}\ \mathrm{s}$

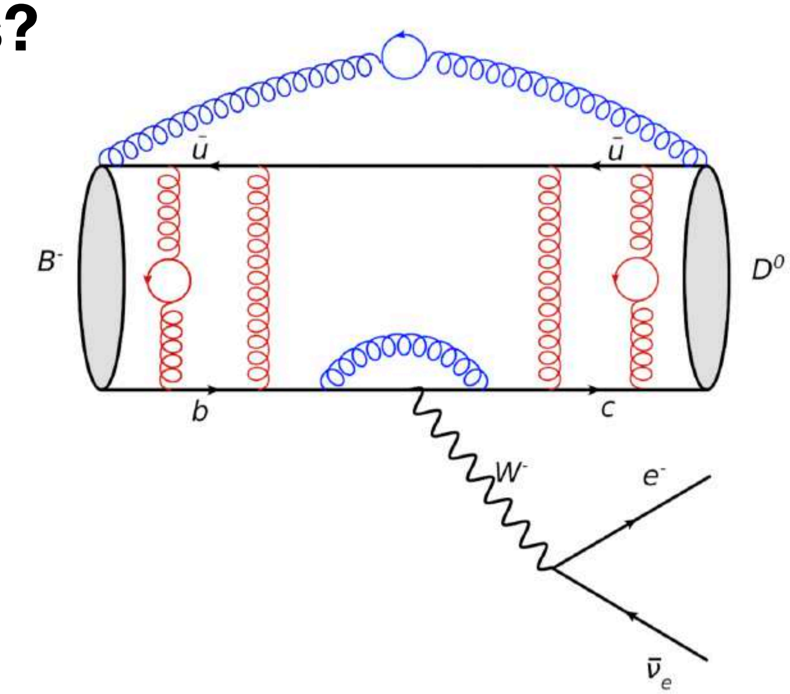
for higher accuracy include QCD corrections

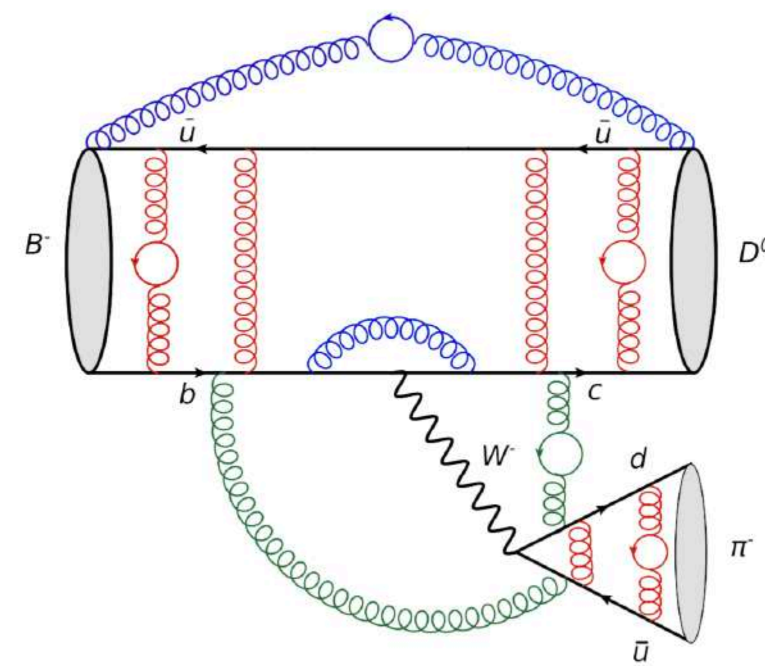
Quark Hadron Duality

Experiment at Hadron Level - Calculation at Quark-Gluon level

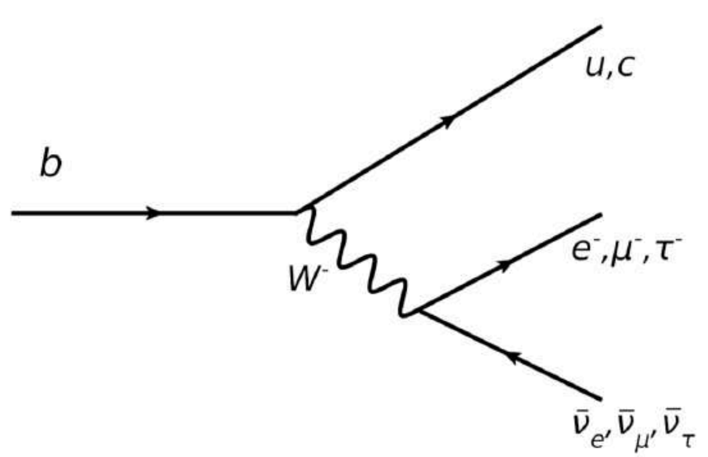
What about hadron decays?

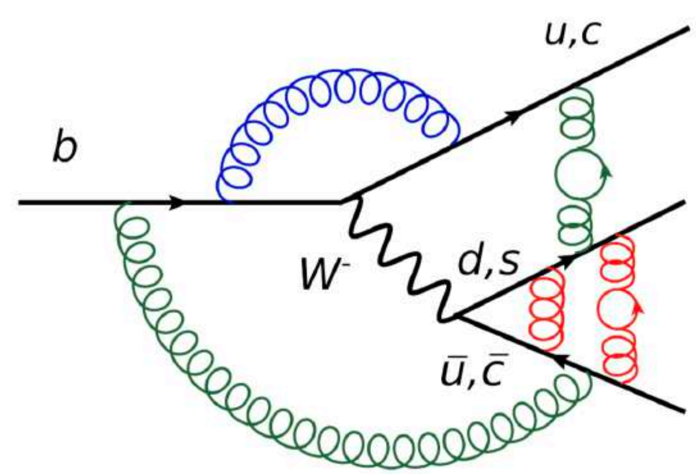






Is the quark level calculation any good approximation at all?
What are the corrections to the quark level calculation?



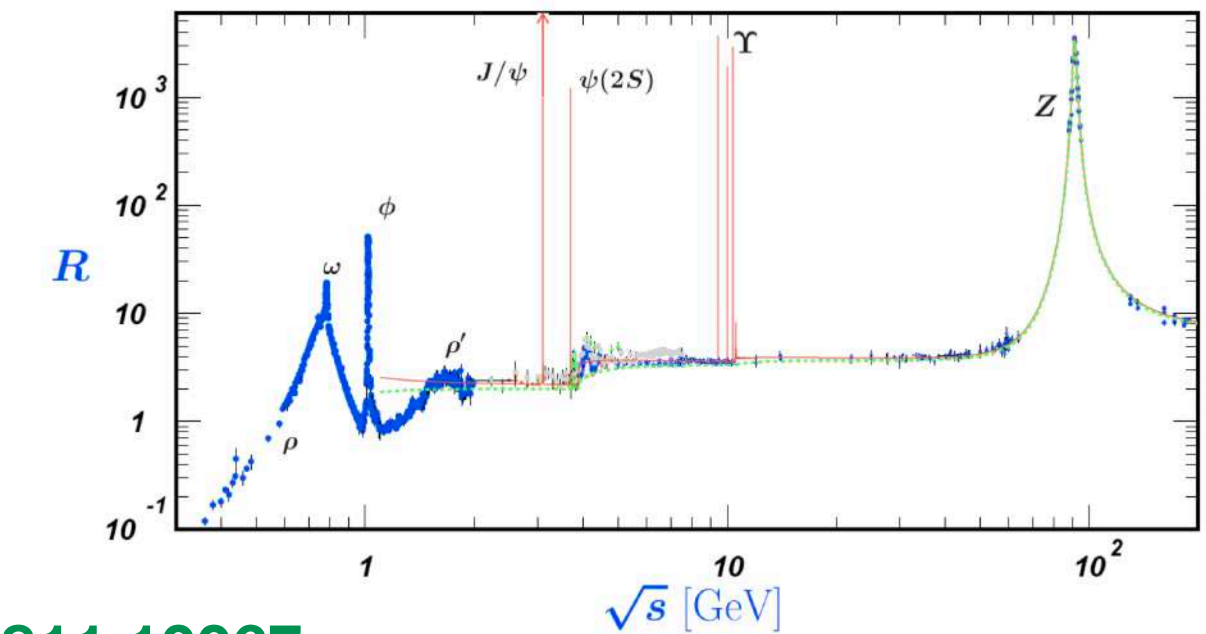


Similar problems arise in many different fields and have a long History

Quark Hadron Duality

Working definition I: QHD states Hadron Level = Quark-Gluon level

- e^+p: Bloom, Gilman 1970/71
- e^+-e^- annihilation:
  Poggio, Quinn, Weinberg 1976



- Hadronic tau decays: e.g. Pich 1811.10067

$$\alpha_s^{(n_f=5)}(M_Z^2)\Big|_\tau - \alpha_s^{(n_f=5)}(M_Z^2)\Big|_Z = 0.0001 \pm 0.0015_\tau \pm 0.0030_Z$$

- Decays of heavy Hadrons
- Physics at the Z-peak
- ....

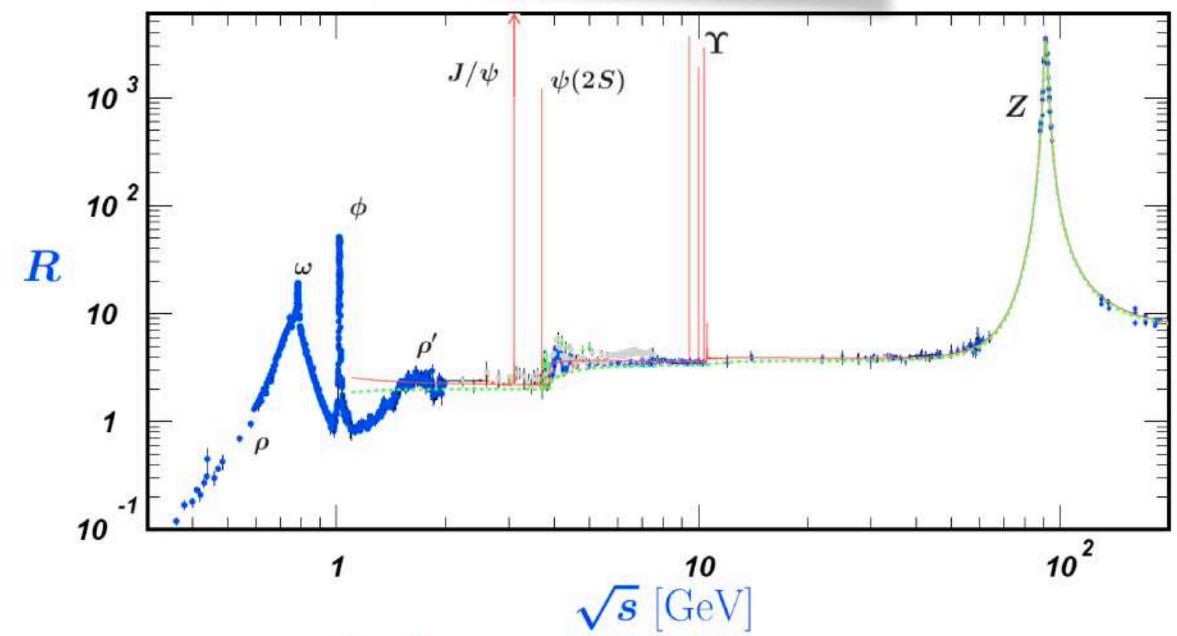
a) Total inclusive decay rates
b) Decay rate differences Gamma_12
c) Inclusive semileptonic decays
d) Exclusive decays

Quark Hadron Duality

Working definition I: QHD states Hadron Level = Quark-Gluon level

- **e^+-e^- annihilation:**
  Poggio, Quinn, Weinberg 1976

$$R = \frac{\sigma_H}{\sigma_{\mu\mu}} = N_c \sum_q \left(\frac{e_q}{e}\right)^2 = 3 \cdot \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = \frac{11}{3}$$



$$\sigma^{\rm tot}\left(e^+ + e^- \to \text{hadrons}\right) = \sigma^{\rm tot}\left(e^+ + e^- \to \text{quarks}\right)$$ **?**

What else should the quarks do except hadronising???

Smeared cross section agrees with calculation of the vacuum polarisation

$$\bar{\sigma}(s) = \frac{\Delta}{\pi}\int_0^\infty \frac{\sigma(s')ds'}{(s-s')^2+\Delta^2} = \frac{1}{2i}\left[\Pi(s+i\Delta) - \Pi(s-i\Delta)\right]$$

Heavy Quark Expansion

Shifman, Voloshin, Khoze; Bigi, Uraltsev, Vainshtein; (1983 -'92)

Decays of heavy quarks are described by the effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_c^q (C_1 Q_1^q + C_2 Q_2^q) - V_p \sum_{j=3} C_j Q_j \right]$$

$$Q_2 = c_\alpha \gamma_\mu (1-\gamma_5) \bar{b}_\alpha \times d_\beta \gamma^\mu (1-\gamma_5) \bar{u}_\beta$$

The total decay rate of a heavy hadron is given by

$$\Gamma(B \to X) = \frac{1}{2m_B} \sum_X (2\pi)^4 \delta^{(4)}(p_B - p_X) |\langle X|\mathcal{H}_{eff}|B\rangle|^2$$

According to the optical theorem this can written as a double insertion of the effective Hamiltonian

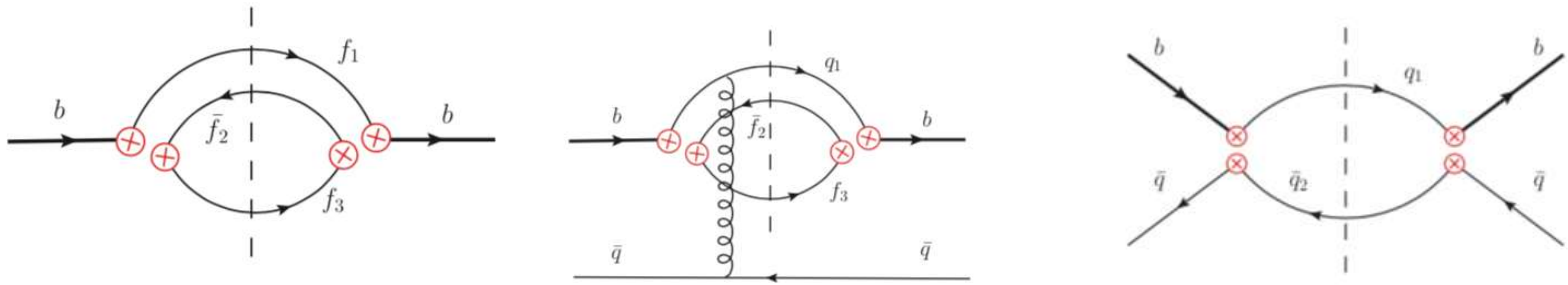
$$\Gamma(B \to X) = \frac{1}{2m_B} \langle B|\mathcal{T}|B\rangle \,, \qquad \mathcal{T} = \mathrm{Im}\, i \int d^4x T \left[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right] \,,$$

Heavy Quark Expansion

Shifman, Voloshin, Khoze; Bigi, Uraltsev, Vainshtein; (1983 -'92)

$$\mathcal{T} = \mathrm{Im}\; i \int d^4x T\left[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right] ,$$

Different Wick contraction give different topologies



Integrating out these diagrams gives the following Taylor expansion in local operators

$$\mathcal{T} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b}\bar{b}b + \frac{c_{5,b}}{m_b^2}\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b + 2\frac{c_{6,b}}{m_b^3}(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma + \ldots \right]$$

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[c_{3,b}\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + \frac{c_{5,b}}{m_b^2}\frac{\langle B|\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{2M_B} + \frac{c_{6,b}}{m_b^3}\frac{\langle B|(\bar{b}q)_\Gamma(\bar{q}b)_\Gamma|B\rangle}{M_B} + \ldots \right]$$

Heavy Quark Expansion

Shifman, Voloshin, Khoze; Bigi, Uraltsev, Vainshtein; (1983 -'92)

In more detail we get

$$\Gamma = \Gamma_0 \langle O_{D=3} \rangle + \Gamma_2 \frac{\langle O_{D=5} \rangle}{m_Q^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6} \rangle}{m_Q^3} + \ldots$$
$$+16\pi^2 \left[\Gamma_3 \frac{\langle O_{D=6} \rangle}{m_Q^3} + \Gamma_4 \frac{\langle O_{D=7} \rangle}{m_Q^4} + \Gamma_5 \frac{\langle O_{D=8} \rangle}{m_Q^5} + \ldots \right]$$

Working definition II of QHDV = deviation from the above framework

- Γ_0 **: free quark decay**
- **Perturbative corrections in Gamma_i**
- **Non-perturbative corrections in matrix elements**
- **There are no 1/mQ corrections**
- Γ_2 **: kinetic and chromomagnetic term**
- $\tilde{\Gamma}_3$ **: Spin-orbit and Darwin term**
- Γ_3 **: Spector effects, 1-loop instead of 2-loops**
- Γ_4 Γ_5 **: 1/mQ corrections to** Γ_3

Old Problems

$$\frac{\tau(B_s)}{\tau(B_d)}^{\text{HQE 1986}} \approx 1\,, \quad \frac{\tau(B^+)}{\tau(B_d)}^{\text{HQE 1986}} \approx 1.1\,, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)}^{\text{HQE 1986}} \approx 0.96$$

Hierarchy of Lifetimes of Charmed and Beautiful Hadrons
Mikhail A. Shifman, M.B. Voloshin (Moscow, ITEP). 1986. 30 pp.
Published in **Sov.Phys.JETP 64 (1986) 698, Zh.Eksp.Teor.Fiz. 91 (1986) 1180-1193**
ITEP-86-83
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Detailed record - Cited by 281 records 250+

Experimental numbers for $\tau(\Lambda_b)$

2003	HFAG	average	1.212 ± 0.052	0.798 ± 0.034
1998	OPAL	$\Lambda_c l$	1.29 ± 0.25	$0.85 \pm 0.16*$
1998	ALEPH	$\Lambda_c l$	1.21 ± 0.11	$0.80 \pm 0.07*$
1995	ALEPH	$\Lambda_c l$	1.02 ± 0.24	$0.67 \pm 0.16*$
1992	ALEPH	$\Lambda_c l$	1.12 ± 0.37	$0.74 \pm 0.24*$

Old Problems

Many theory paper appeared

Some claiming HQE fails

FAILURE OF LOCAL DUALITY IN INCLUSIVE
NON-LEPTONIC HEAVY FLAVOUR DECAYS

G. Altarelli

Theoretical Physics Division, CERN, CH-1211 Geneva 23 and
Dipartimento di Fisica, Terza Università di Roma, Roma

G. Martinelli, S. Petrarca and F. Rapuano

Dip. di Fisica dell'Università *La Sapienza* and
INFN, Sez. di Roma I
P.le A. Moro 2, 00185 Roma, Italy

arXiv:hep-ph/9604202v3 5 Apr 1996

ABSTRACT

We argue that there is strong experimental evidence in the data of *b*- and *c*-decays that the pattern of power suppressed corrections predicted by the short distance expansion, the heavy quark effective theory and the assumption of local duality is not correct for the non-leptonic inclusive widths. The data indicate instead the presence of $1/m$ corrections that should be absent in the above theoretical framework. These corrections can be simply described by replacing the heavy quark mass by the mass of the decaying hadron in the m^5 factor in front of all the non-leptonic widths.

Nature (or experimentalists) might be nasty

Experiment in 1996 shows

$$\Gamma^{\rm NL} = \frac{G_F^2 m_{\rm Meson}^5}{192\pi^3} \quad \text{vs.} \quad \Gamma^{\rm NL} = \frac{G_F^2 m_{\rm b}^5}{192\pi^3}$$

↓ **Works** ↓ **Works Not**

Old Problems

Theoretical numbers for $\tau(\Lambda_b)$

Many theory paper appeared

Some claiming to be able to predict experiment within the HQE while some just see a discrepancy with experiment

Year	Author	$\tau(\Lambda_b)/\tau(B_d)$
2007	Tarantino	0.88 ± 0.05
2004	Petrov et al.	0.86 ± 0.05
2003	Tarantino	0.88 ± 0.05
2002	Rome	0.90 ± 0.05
2000	Körner,Melic	$0.81 ... 0.92$
1999	Guberina,Melic,Stefanic	0.90
1999	diPierro, Sachrajda, Michael	0.92 ± 0.02
1999	Huang, Liu, Zhu	0.83 ± 0.04
1996	Colangelo, deFazio	> 0.94
1996	Neubert,Sachrajda	$" > 0.90"$
1992	Bigi, Blok, Shifman, Uraltsev, Vainshtein	$> 0.85 ... 0.90$
x	only$1/m_b^2$	0.98

Colour coding:

- Wilson coefficient
- Matrix element of dimension 6 operator
- Numerical update

Old Problems

Experimental numbers for $\tau(\Lambda_b)$

$$\frac{\tau(B_s)}{\tau(B_d)}^{\mathrm{HQE\,1986}} \approx 1\,, \quad \frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{HQE\,1986}} \approx 1.1\,, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)}^{\mathrm{HQE\,1986}} \approx 0.96$$

Year	Exp	Decay	$\tau(\Lambda_b)$ [ps]	$\tau(\Lambda_b)/\tau(B_d)$
2011	HFAG	average	1.425 ± 0.032	0.938 ± 0.022
2010	CDF	$J/\psi\Lambda$	1.537 ± 0.047	1.020 ± 0.031
2009	CDF	$\Lambda_c + \pi^-$	1.401 ± 0.058	0.922 ± 0.038
2007	D0	$\Lambda_c \mu\nu X$	1.290 ± 0.150	$0.849 \pm 0.099*$
2007	D0	$J/\psi\Lambda$	1.218 ± 0.137	$0.802 \pm 0.090*$
2006	CDF	$J/\psi\Lambda$	1.593 ± 0.089	1.049 ± 0.059
2004	D0	$J/\psi\Lambda$	1.22 ± 0.22	0.87 ± 0.17
2003	HFAG	average	1.212 ± 0.052	0.798 ± 0.034
1998	OPAL	$\Lambda_c l$	1.29 ± 0.25	$0.85 \pm 0.16*$
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1992	ALEPH	$\Lambda_c l$	1.12 ± 0.37	$0.74 \pm 0.24*$

As soon as hadronic final states could be investigated, the experimental values changed dramatically

1.425 ps is $4.1\sigma (4.1 * 0.052)$ above 1.212 ps

Old Problems have vanished

Status in 2019

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 0.935 \pm 0.054$$

AL 2014
Uraltsev Memorial Book

Λ_b	**1.471 ± 0.009 ps**	$\Lambda_b/B^0 = 0.969 \pm 0.006$

4.9 sigma above 2003 average!!!

keep this in mind when discussing experimental anomalies

HFLAV 2019

In the 90ies there were also other problems - all have disappeared

- Baffling Semileptonic Branching Ratio
- Missing Charm Puzzle

Open questions

- What happens if we are not summing over all states, e.g. Delta Gamma_s

What could go wrong?

OPE is valid in the Euclidean region = large complex energies
Physics = real energies
=> Analytic continuation necessary

Problem: Series is truncated in alpha_s and 1/M_Q
Non-perturbative 1/M_Q and exponential terms might exist

$$\exp[-m_b/\Lambda]$$

that are not contained in a Taylor Expansion
=> 1/M_Q terms and oscillatory terms after analytic continuation

Global quark hadron duality: e.g. semi-leptonic decays, tau decays
phase space integration over lepton momentum = smearing

Local quark hadron duality: non-leptonic decays

Violations of QHD:
A. 1/m_Q terms arise
B. Oscillatory terms arise

Theoretical approaches to tackle QHD

Theoretical solution of whether QHD is violated or not requires
a full solution of QCD
and a subsequent comparison to predictions of the HQE….
clearly impossible

=> Study simplified models of nature

1. **SV limit** $N_c \to \infty \quad m_b, m_c \ll m_b - m_c \ll \Lambda_{QCD}$

1995 Boyd, Grinstein, Manohar: Duality holds for semi-leptonic decays

SV-limit

arXiv.org > hep-ph > arXiv:hep-ph/0304202v1

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High Energy Physics - Phenomenology

Explicit Quark-Hadron Duality Violations in B-Meson Decays

Benjamin Grinstein, Michael Savrov

(Submitted on 22 Apr 2003 (this version), **latest version 29 Apr 2003** *(v2))*

We consider the weak decay of heavy mesons in QCD. We compute the inclusive hadronic decay rate in leading order in the large N_c expansion, with masses chosen to insure the final state mesons recoil slowly (the SV limit). We find, by explicit computation, violations to quark-hadron duality at order 1/M in the heavy mass expansion. The violation to duality is linear in the slope of the form factor for the associated semileptonic decay. Differences in slopes of form factors may help understand the puzzle of lifetimes of b-hadrons.

Comments:	17 pages, no figures, latex/revtex4
Subjects:	**High Energy Physics - Phenomenology (hep-ph)**
Report number:	UCSD/PTH 03-05
Cite as:	arXiv:hep-ph/0304202
	(or arXiv:hep-ph/0304202v1 for this version)

Bibliographic data

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Submission history

From: Benjamin Grinstein [view email]
[v1] Tue, 22 Apr 2003 05:10:07 UTC (16 KB)
[v2] Tue, 29 Apr 2003 21:34:58 UTC (0 KB)

SV-limit

arXiv.org > hep-ph > arXiv:hep-ph/0304202

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High Energy Physics - Phenomenology

Explicit Quark-Hadron Duality Violations in B-Meson Decays

Benjamin Grinstein, Michael Savrov

(Submitted on 22 Apr 2003 (v1), last revised 29 Apr 2003 (this version, v2))

Duality is not violated at order Delta/M once j=3/2 and j=1/2+ states are properly accounted for.

Comments:	Paper withdrawn by authors, due to crucial omission of higher resonances
Subjects:	**High Energy Physics - Phenomenology (hep-ph)**
Report number:	UCSD/PTH 03-05
Cite as:	**arXiv:hep-ph/0304202**
	(or **arXiv:hep-ph/0304202v2** for this version)

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Theoretical approaches to tackle QHD

Theoretical solution of whether QHD is violated or not requires
a full solution of QCD
and a subsequent comparison to predictions of the HQE….
clearly impossible and unnecessary

=> Study simplified models of nature

- **SV limit** $N_c \to \infty \quad m_b, m_c \ll m_b - m_c \ll \Lambda_{QCD}$

  1995 Boyd, Grinstein, Manohar: Duality holds for semi-leptonic decays
  2003 Grinstein, Savrov: also for non-leptonic ones

- **Instanton based models**

- **Resonance based models**

- **'tHooft model: D = 1+1, N_c = infinity** $\mathcal{L}_{\text{'t Hooft}} = -\frac{N_c}{4\pi\Lambda^2}\mathrm{tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + i\overline{\psi}\not{D}\psi - m_q\overline{\psi}\psi\,,$

9708396 Grinstein, Lebed: small 1/M_Q correction for non-leptonic decays
9805241: Bigi, Shifman, Uraltsev, Vainshtein: no 1/M_Q terms, but tiny oscillatory ones
9805404 Grinstein, Lebed: QHD - not good for annihilation contribution
9903258: Bigi, Uraltsev: QHD works well for Pauli-interference
0006346: Lebed, Uraltsev: impressive agreement with HQE for semi-leptonic decays
0106205 Grinstein: 1/M_Q^2 corrections, if smeared -> QHD violation?

Shut up and calculate in the real world

What is the state of the art of the HQE?
How does it compare to Experiment ?

$$\Gamma = \Gamma_0 \langle O_{D=3} \rangle + \Gamma_2 \frac{\langle O_{D=5} \rangle}{m_Q^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6} \rangle}{m_Q^3} + \ldots$$
$$+16\pi^2 \left[\Gamma_3 \frac{\langle O_{D=6} \rangle}{m_Q^3} + \Gamma_4 \frac{\langle O_{D=7} \rangle}{m_Q^4} + \Gamma_5 \frac{\langle O_{D=8} \rangle}{m_Q^5} + \ldots \right]$$

LIFETIMES

Mark Williams
@QuarkWilliams

Following

How much can I trust theoretical predictions? Finally the star-based rating system I've been waiting for! Thanks @alexlenz42! arxiv.org/pdf/1809.09452…

A + for each independent calculation
At most ++
At most +++ for <>: 2 lattice, 1 sum rule
Punishment: A - - for no <Q6>
A 0 for quark model et al for <Q6>

Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6}\rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7}\rangle$	Σ
$\tau(B^+)/\tau(B_d)$	++	++	0	+	++	0	0	** (7+)
$\tau(B_s)/\tau(B_d)$	++	++	0	$\frac{+}{2}$	++	0	0	** (6.5+)
$\tau(\Lambda_b)/\tau(B_d)$	++	$\frac{+}{2}$	0	$\frac{+}{2}$	+	0	0	** (4+)
$\tau(b-baryon)/\tau(B_d)$	++	0	0	0	+	0	0	* (3+)
$\tau(B_c)$	+	0	0	+	0	0	0	* (2+)
$\tau(D^+)/\tau(D^0)$	++	++	0	+	++	0	0	** (7+)
$\tau(D_s^+)/\tau(D^0)$	++	++	0	$\frac{+}{2}$	++	0	0	** (6.5+)
$\tau(c-baryon)/\tau(D^0)$	++	0	0	0	+	0	0	* (3+)

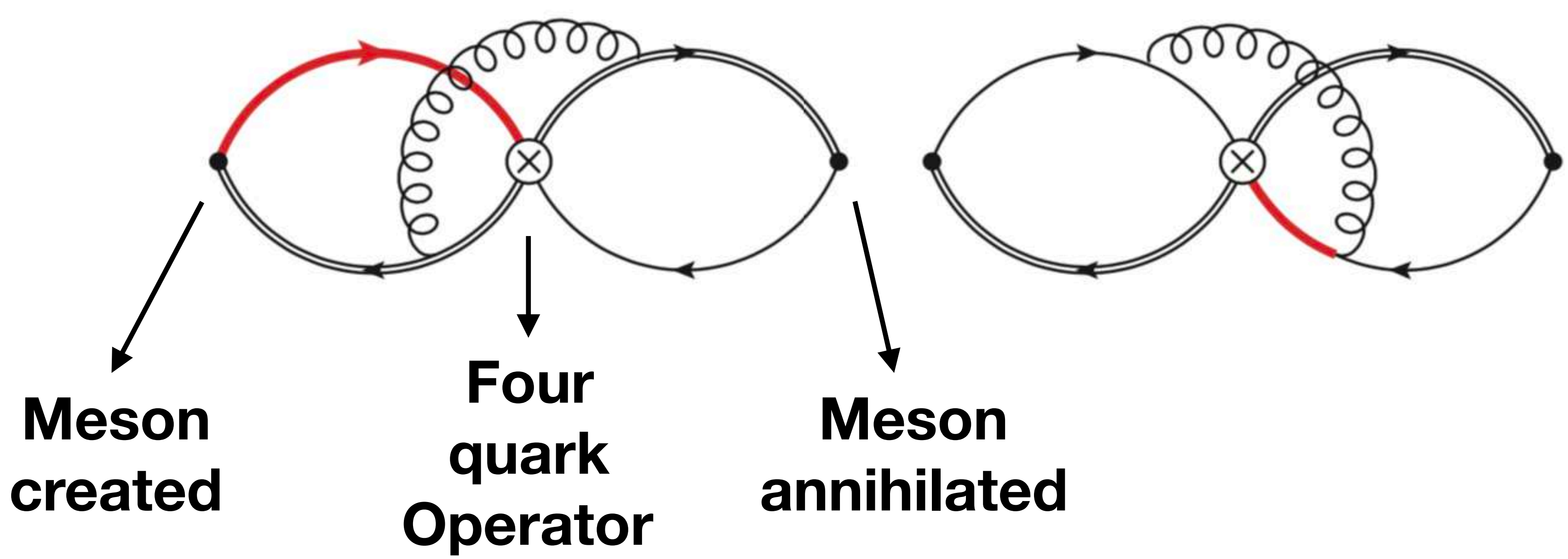
Hai-Yang Cheng 1807.00916

****: 12-15 *** 8 -11.5 **: 4-7.5 *: 2-3.5

LIFETIMES

Most recent development: Determination of D=6 matrix elements in 2017

3-loop HQET sum rules for B+/Bd and D+/D0:



So far only preliminary lattice studies from 2001 and earlier and preliminary sum rule studies from the 90ies

Up to date lattice studies would be very desirable

Claim: This method is competitive to lattice - see mixing case

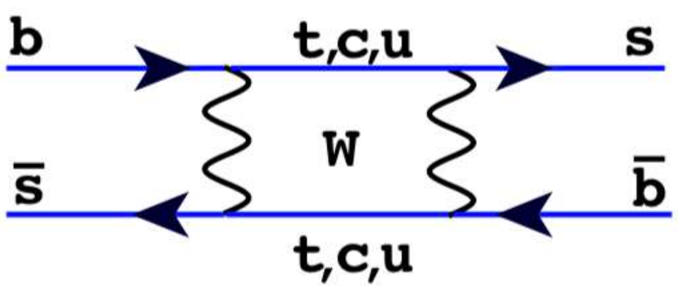
Mass difference ΔM_q

Experiment.: HFLAV 2019

$\Delta m_s = 17.757 \pm 0.021\ \mathrm{ps}^{-1}$

$\Delta m_d = 0.5064 \pm 0.0019\ \mathrm{ps}^{-1}$

Theory



CKM

Inami-Lim

Buras Jamin Weisz

$$M_{12}^s = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s} \hat{\eta}_B$$

In the SM one operator:

$$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta$$

$$\langle Q \rangle \equiv \langle B_s^0 | Q | \bar{B}_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu)$$

Non-perturbative theory input:

1) **Lattice:** ETM, FNAL-MILC, RBC-UKQCD, HPQCD
2) **Sum rules:** Siegen, Durham

Mass difference ΔM_q

Mixing Operators Delta B = 2

$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma_5) s_i \times \bar{b}_j \gamma^\mu (1-\gamma_5) s_j \quad \Big\} \Delta\Gamma_s^{SM}$$

$$Q_2 = \bar{b}_i (1-\gamma_5) s_i \times \bar{b}_j (1-\gamma_5) s_j$$

$$Q_3 = \bar{b}_i (1-\gamma_5) s_j \times \bar{b}_j (1-\gamma_5) s_i$$

$$Q_4 = \bar{b}_i (1-\gamma_5) s_i \times \bar{b}_j (1+\gamma_5) s_j$$

$$Q_5 = \bar{b}_i (1-\gamma_5) s_j \times \bar{b}_j (1+\gamma_5) s_i$$

$\Big\}$ $\Delta\Gamma_s^{SM}$ & $\Delta\Gamma_s^{BSM}$

Γ_3 (for Q_1–Q_3), Γ_4 (for Q_4–Q_5)

Parameterisation in terms of decay constants and Bag parameter

$$\langle B_s | Q_1 | \bar{B}_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B_1$$

$$\langle B_s | Q_2 | \bar{B}_s \rangle = -\frac{5}{3} \left[\frac{M_{B_s}}{m_b + m_s}\right]^2 M_{B_s}^2 f_{B_s}^2 B_2$$

$$\langle B_s | Q_3 | \bar{B}_s \rangle = \frac{1}{3} \left[\frac{M_{B_s}}{m_b + m_s}\right]^2 M_{B_s}^2 f_{B_s}^2 B_3$$

$$\langle B_s | Q_4 | \bar{B}_s \rangle = \left(2\left[\frac{M_{B_s}}{m_b + m_s}\right]^2 + \frac{1}{3}\right) M_{B_s}^2 f_{B_s}^2 B_4$$

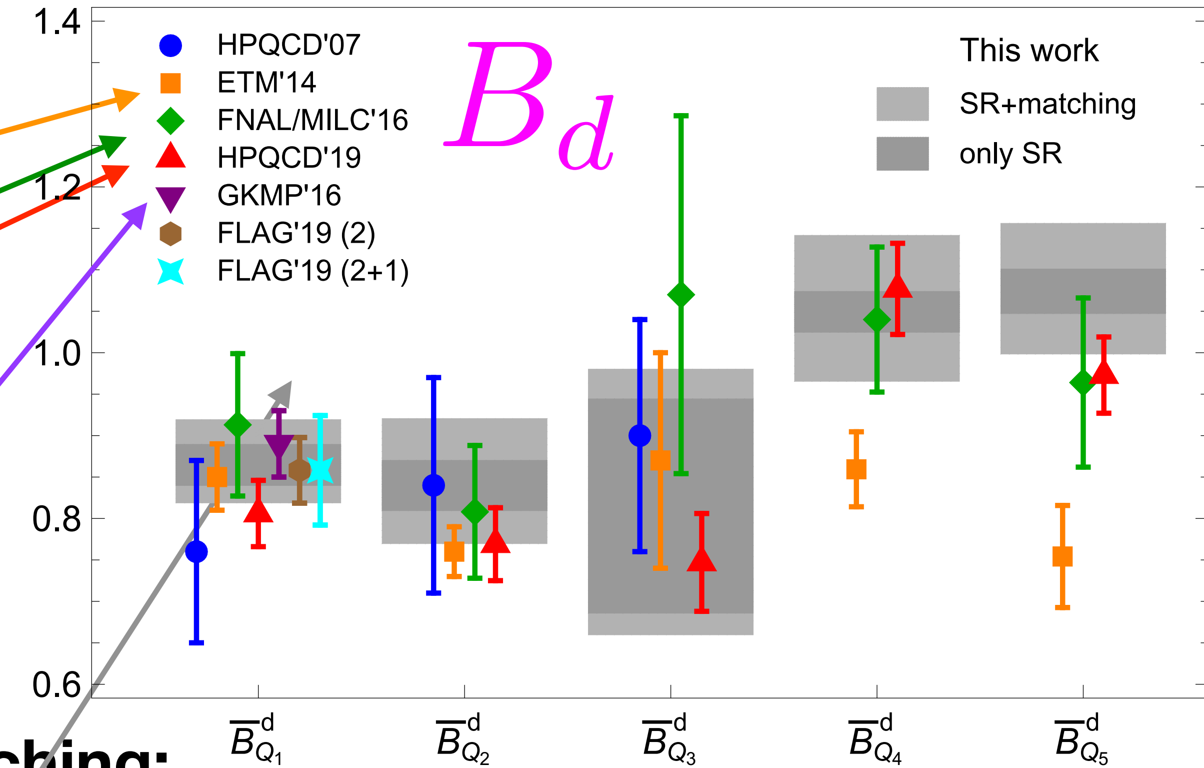
$$\langle B_s | Q_5 | \bar{B}_s \rangle = \left(\frac{2}{3}\left[\frac{M_{B_s}}{m_b + m_s}\right]^2 + 1\right) M_{B_s}^2 f_{B_s}^2 B_5$$

f_{B_s}: • lattice

B_i: • HQET-SR • lattice

Non-perturbative input for ΔM_q

Plot by Thomas Rauh



B_d-mixing

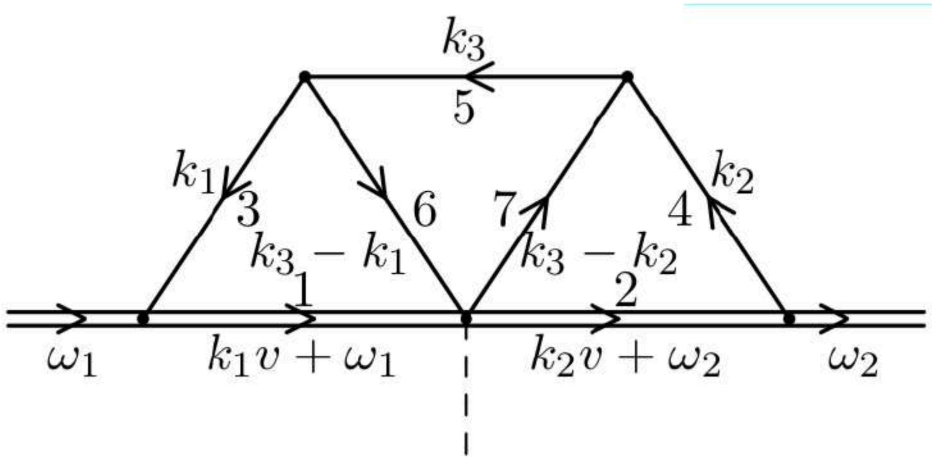
1. **Lattice**
   * **ETM 1308.1851**
   * **FNAL-MILC 1602.03560**
   * **HPQCD 1907.01025**

2. **HQET-sum rules: 3-loop + part of NNLO matching:**
   ***Siegen: Grozin, Klein, Mannel, Pivovarov 1606.06054, 1706.05910, 1806.00253**
   ***Durham: Kirk (Rome), AL, Rauh (Bern) 1711.02100**

Three-loop HQET vertex diagrams for B^0–$\bar{B}^0$ mixing

Andrey G. Grozin and Roman N. Lee

arXiv:0812.4522v2



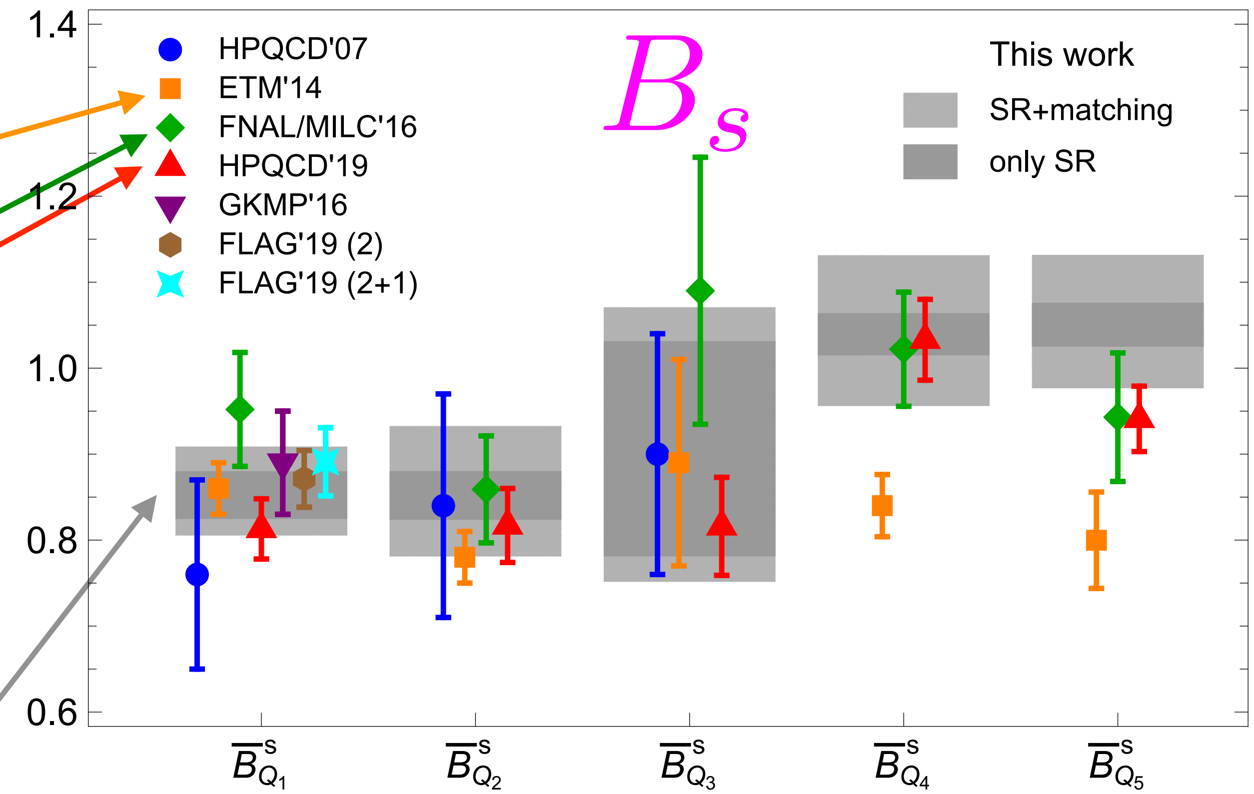
The various NLO contributions:

- Perturbative contribution (3-loop)
  $\Delta B_{PT} = -0.10 \pm 0.02 \pm 0.03$
  A.Grozin,R.Klein,ThM,AAP, Phys.Rev. D94, 034024 (2016)
- Quark condensate contribution (2-loop)
  $\Delta B_q = -0.002 \pm 0.001$
  A.Grozin,R.Klein,ThM,AAP, Phys.Rev. D94, 034024 (2016)
- Other condensates (tree-level+2-loop gluon cond)
  $\Delta B_{nonPT} = -0.006 \pm 0.005$
  ThM, B.D. Pecjak, AAP, Eur.Phys.J. C71 (2011) 1607

Total $\Delta B = -0.11 \pm 0.04 \pm 0.03$

Non-perturbative input for ΔM_q

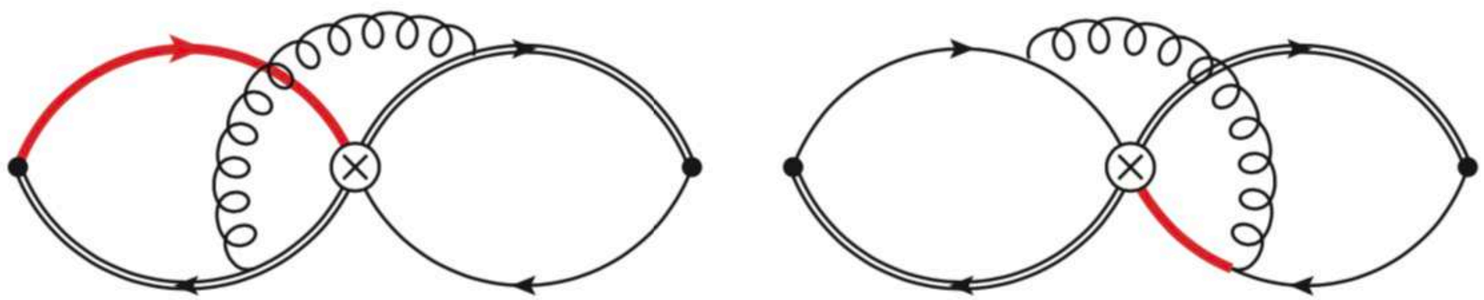
Plot by Thomas Rauh



B_s-mixing

1. **Lattice**
   * **ETM 1308.1851**
   * **FNAL-MILC 1602.03560**
   * **HPQCD 1907.01025**

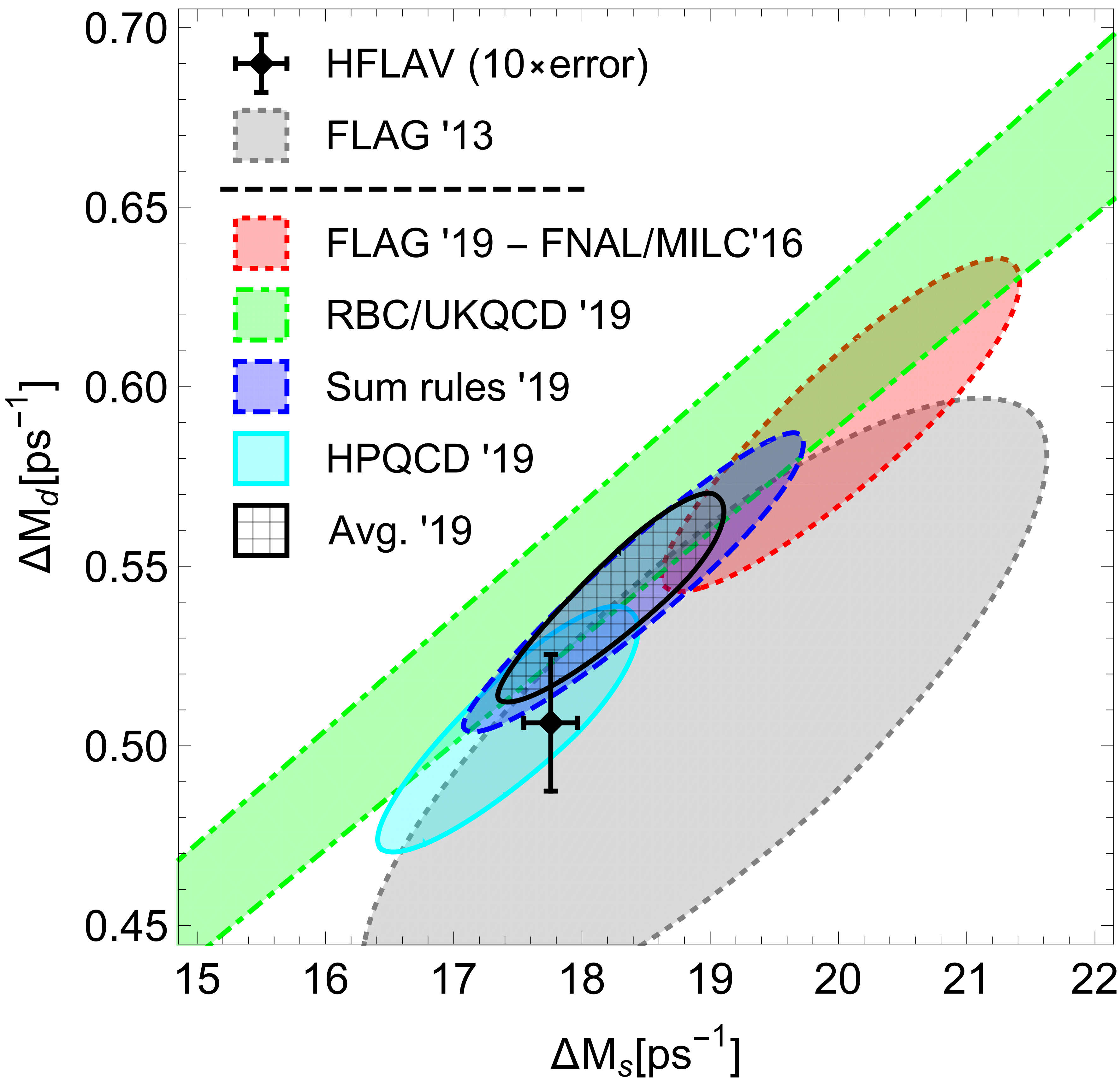
2. **HQET-sum rules: 3-loop + NLO matching:**
   ***Durham: King, AL, Rauh (Bern) 1904.00940**



$$r^{(0)}_{\tilde{Q}_1} = 8 - \frac{a_2}{2} - \frac{8\pi^2}{3}$$

$$r^{(2)}_{\tilde{Q}_1} = \frac{1}{1+x^2}\left[\frac{(1-x)^2 a_2}{4} + \frac{2\pi^2(1-4x+x^2)}{3} + 2x\psi(x)\left(2 + \frac{1+x}{1-x}\ln(x)\right)\right.$$
$$\left. + \begin{cases} -\frac{2(6+6x-x^2+2x^3)}{3} + 2(2-4x+x^2)\ln(x) - 4(1-x^2)\mathrm{Li}_2(1-1/x), & x \le 1, \\ -\frac{2(2-x+6x^2+6x^3)}{3x} - 2(1-4x+2x^2)\ln(x) + 4(1-x^2)\mathrm{Li}_2(1-x), & x > 1, \end{cases}\right]$$

Sum rules rule



Very active field:

- Flag 19: mostly FNAL-MILC (2/16)
- RBC-UK: 12-18
- Sum rules: Durham 4/19 (based on Siegen 16-18, Durham 17)
- HPQCD: 07/19

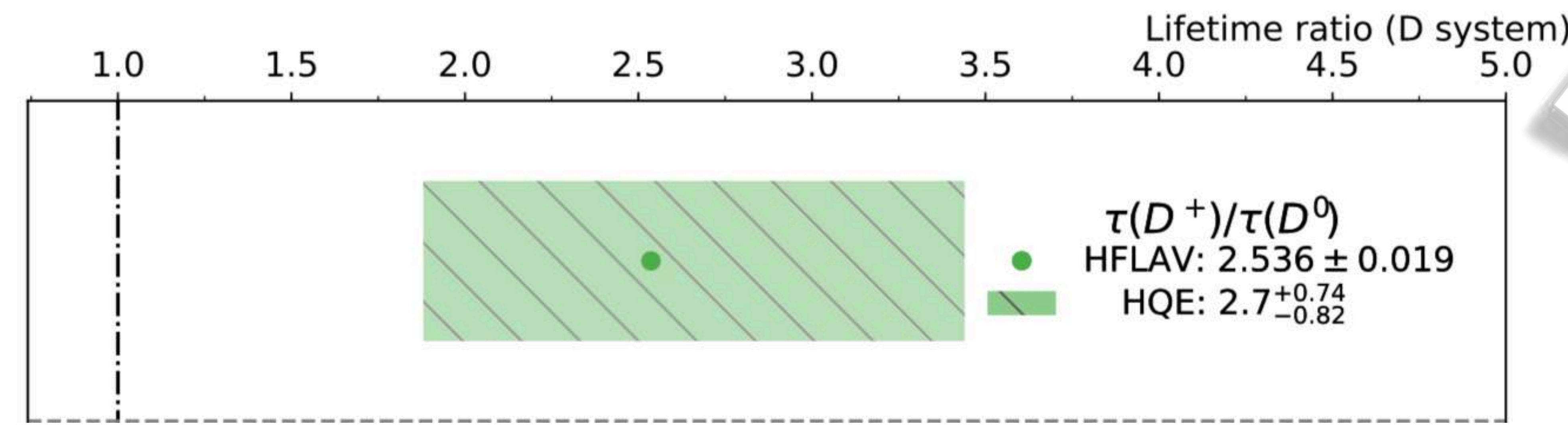
Method is very successful in mixing - What results do we get with this method for lifetimes?

Charm Lifetimes

$\Lambda/m_c \approx 3\Lambda/m_b$ - could still give some reasonable estimates!

Look in systems without GIM cancellation: D-lifetimes

NEW 3-loop sum rules



$$\frac{\tau(D^+)}{\tau(D^0)} = 2.7 = 1 + 16\pi^2\,(0.25)^3\,(1 - 0.34)$$

Kirk, AL, Rauh 1711.02100

pert. NLO-QCD:

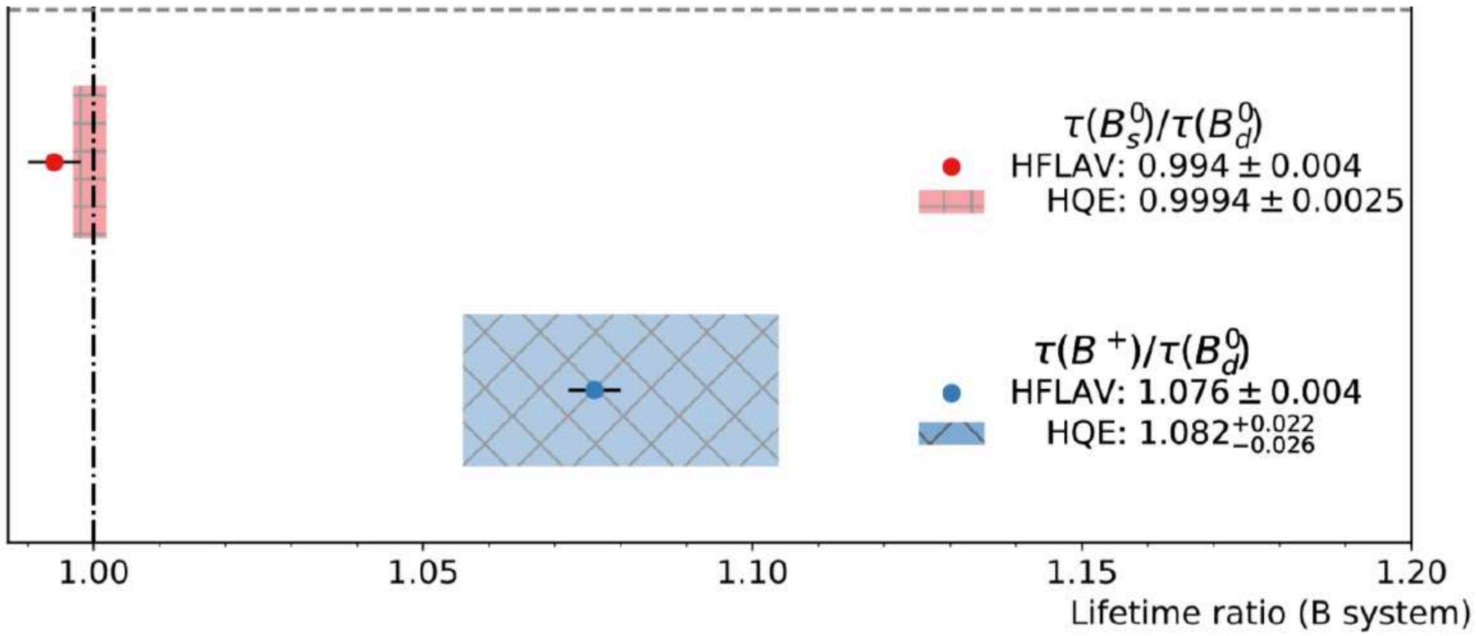
AL, Rauh 1305.3588

Expansion parameter for HQE in charm = 0.3
not a back of envelope statement, but real calculations

d=6 calculated with sum rules
lattice confirmation urgently needed

d=7 estimated in vacuum insertion approximation
do sum rule/lattice

BEAUTY LIFETIMES



Dim 6 matrix elements determined with 3loop HQET sum rules
Lattice confirmation very welcome

Kirk, AL, Rauh 1711.02100

Amazing cancellations in the Bs/Bd system

$$\frac{\tau(B_s)}{\tau(B_d)} = \frac{\Gamma_b + \delta\Gamma_{B_d}}{\Gamma_b + \delta\Gamma_{B_s}} = 1 + \left(\delta\Gamma_{B_d} - \delta\Gamma_{B_s}\right)\tau(B_s)\,.$$

Leads to an unexpected sensitivity to

- **higher orders in the HQE**

$$\Gamma = \Gamma_0\langle O_{D=3}\rangle + \Gamma_2\frac{\langle O_{D=5}\rangle}{m_Q^2} + \tilde{\Gamma}_3\frac{\langle \tilde{O}_{D=6}\rangle}{m_Q^3} + \ldots$$
$$+16\pi^2\left[\Gamma_3\frac{\langle O_{D=6}\rangle}{m_Q^3} + \Gamma_4\frac{\langle O_{D=7}\rangle}{m_Q^4} + \Gamma_5\frac{\langle O_{D=8}\rangle}{m_Q^5} + \ldots\right]$$

in progress

- **invisible Bs decays at the permille level, e.g.**

$$Br(B_s \to \tau^+\tau^-) < 6.8\cdot 10^{-3}\ \text{LHCb}$$
$$Br(B_d \to \tau^+\tau^-) < 2.1\cdot 10^{-3}\ \text{LHCb}$$

$$\frac{\tau(B_s)}{\tau(B_d)} \approx 1 + \delta\Gamma_{B_d}^{\rm SM}\tau(B_s) - \delta\Gamma_{B_s}^{\rm SM}\tau(B_s)$$
$$+ Br(B_d \to X)^{\rm BSM} - Br(B_s \to Y)^{\rm BSM},$$

HEAVY HADRON LIFETIMES

1/Lifetime = total decay rate = Sum over all possible final states

Comparison of experiment and HQE

- **Agrees at sub-percent level for Bs/Bd**
- **Agrees at 2 percent level for B+/Bd**
- **Agrees at 5 percent level for Lambda_b/Bd**
- **Agrees at 70 percent level for D+/D0**

Precision mostly limited by theory

- **Can be improved by about a factor of two by sum rules**
- **Can be improved considerably by lattice**
- **Can extended to more hadron systems**

What about decay rate differences of B_s?
= Sum over final states common to Bs and barBs

Decay rate difference $\Delta\Gamma_s$

Calculation is more difficult than mass difference - use Heavy Quark Expansion

$$\Gamma_{12} \; = \; \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots \, .$$

Each term can be split up into a perturbative part and non-perturbative matrix elements

$$\Gamma_i \; = \; \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi}\Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2}\Gamma_i^{(2)} + \ldots , \right] \langle O^{d=i+3} \rangle$$

$$R_2 = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$R_3 = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

Status of theory predictions

known → Q_1, Q_2, Q_3, Q_4, Q_5

HP-QCD 1910.00970 → R_2, R_3

$$Q_1, Q_2, Q_3 \quad Q_4, Q_5, R_2, R_3$$

$Obs.$	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	$\sum$
Γ_{12}^s	$++$	$++$	$\frac{\pm}{2}$	$+++$	$++$	0	$+$	$10.5 + (***)$
Γ_{12}^d	$++$	$++$	0	$+++$	$++$	0	$+$	$10 + (***)$

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- **Decay constants cancel completely**
- **Bag parameter cancel largely**

SM predictions

$$\Delta\Gamma_s^{\rm exp} = (0.088 \pm 0.006)\,{\rm ps}^{-1},$$
$$\Delta\Gamma_s^{\rm SR} = (0.091^{+0.022}_{-0.030})\,{\rm ps}^{-1}$$
$$= (0.091 \pm 0.020\,({\rm had.})^{+0.008}_{-0.021}\,({\rm scale})^{+0.002}_{-0.005}\,({\rm param.}))\,{\rm ps}^{-1}.$$

- **Good agreement**
- **Experiment about 4 times more precise**

- **Strong test of HQE**
- **Violation of Quark hadron duality must be small**
- **Dim 7 operator have to be determined**
- **NNLO-QCD corrections have to be determined**

$$\Delta\Gamma_d^{\rm exp} = (\,-1.3 \pm 6.6) \cdot 10^{-3}\,{\rm ps}^{-1},$$
$$\Delta\Gamma_d^{\rm SR} = (2.6^{+0.6}_{-0.9}) \cdot 10^{-3}\,{\rm ps}^{-1}$$
$$= (2.6 \pm 0.6\,({\rm had.})^{+0.2}_{-0.6}\,({\rm scale})^{+0.1}_{-0.2}\,({\rm param.})) \cdot 10^{-3}\,{\rm ps}^{-1},$$

- **Might be a solution to the D0 di-muon asymmetry**
- **Experimental number needed**

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- **Decay constants cancel completely**
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SM predictions

$$\Delta\Gamma_s^{\rm exp} = (0.088 \pm 0.006)\,{\rm ps}^{-1},$$
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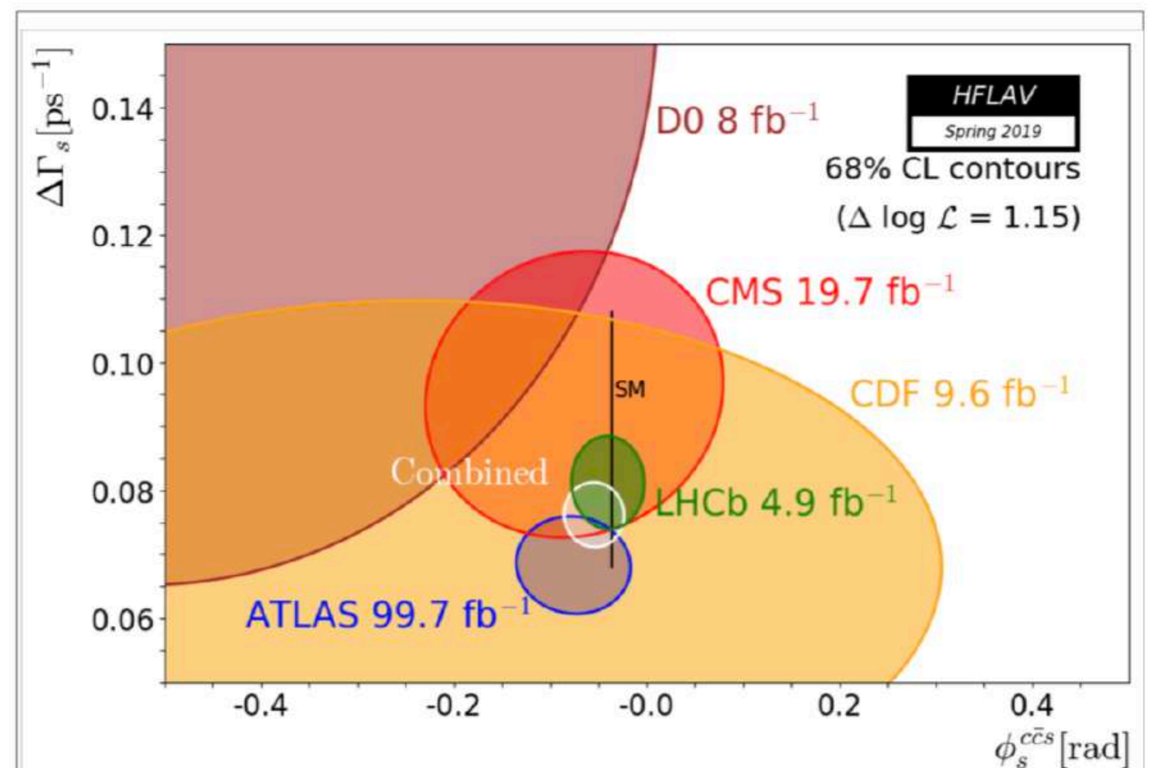
$$= (0.091 \pm 0.020\,({\rm had.})^{+0.008}_{-0.021}\,({\rm scale})^{+0.002}_{-0.005}\,({\rm para$$

- **Good agreement**
- **Experiment about**

- **Strong tes**
- **Violation (**
- **Dim 7 ope**
- **NNLO-QC**

$$\Delta\Gamma_d^{\rm exp} = (\,-1.3 \pm 6.6) \cdot 10^{-3}\,{\rm ps}^{-1},$$
$$\Delta\Gamma_d^{\rm SR} = (2.6^{+0.6}_{-0.9}) \cdot 10^{-3}\,{\rm ps}^{-1}$$
$$= (2.6 \pm 0.6\,({\rm had.})^{+0.2}_{-0.6}\,({\rm scale})^{+0.1}_{-0.2}\,({\rm param.})) \cdot 1$$

- **Might be a solutio**
- **Experimental num**



Very preliminary HFLAV combination

$$\varphi_s = -0.054 \pm 0.021\ {\rm rad}$$
$$\Delta\Gamma_s = 0.0762 \pm 0.0034\ {\rm ps}^{-1}$$

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

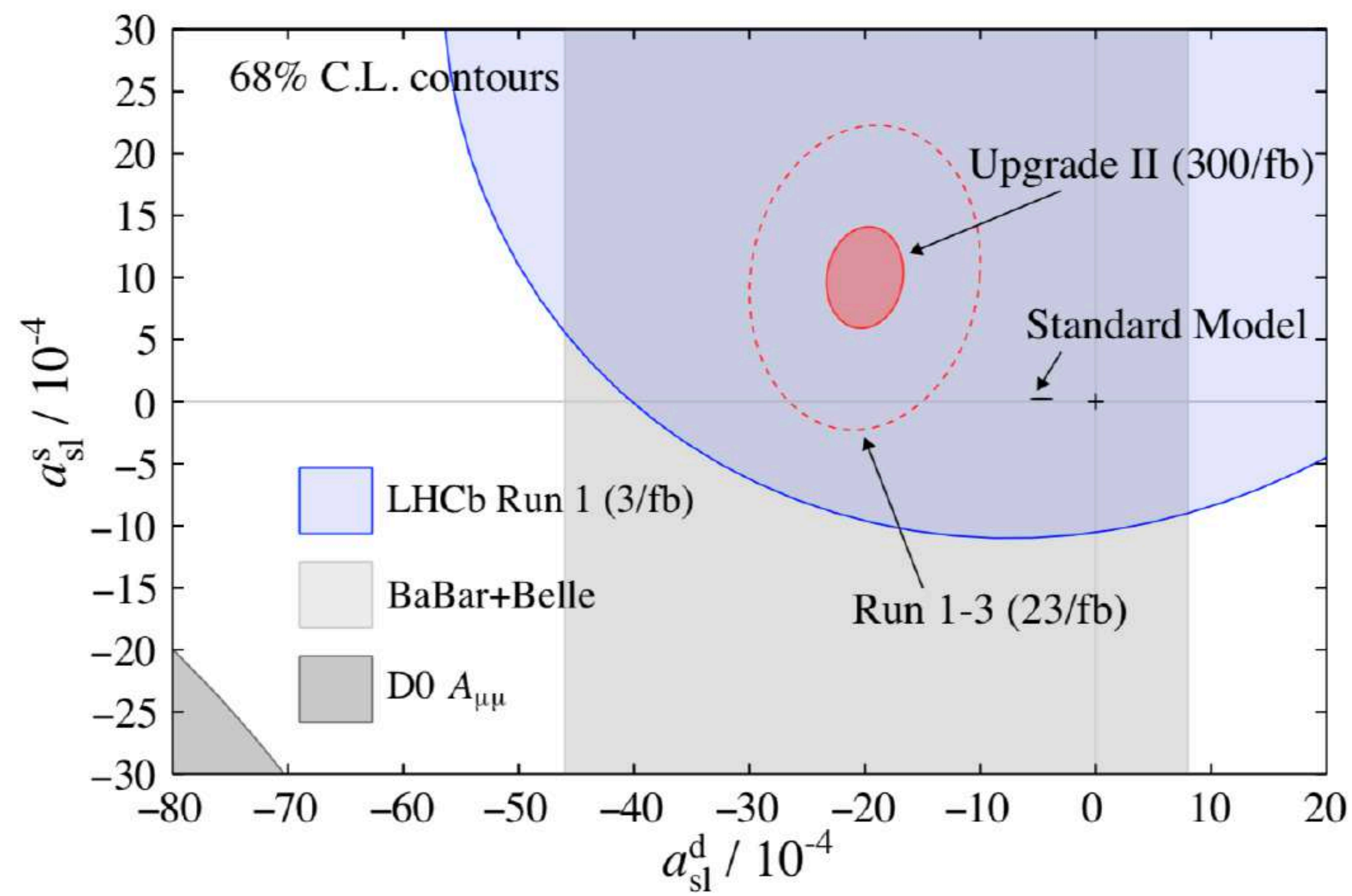
- **Decay constants cancel completely**
- **Bag parameter cancel largely**

SM predictions

$$a_{\text{fs}}^{d,\text{SM},2015} = (-4.7 \pm 0.6) \cdot 10^{-}$$

$$a_{\text{fs}}^{s,\text{SM},2015} = (2.22 \pm 0.27) \cdot 10^{-5}$$

- **Very sensitive to BSM effects!**
- **Experimental number needed**



HQE Predictions

1/Lifetime = total decay rate = Sum over all possible final states

Comparison of experiment and HQE

- **Agrees at sub-percent level for Bs/Bd**
- **Agrees at 2 percent level for B+/Bd**
- **Agrees at 5 percent level for Lambda_b/Bd**
- **Agrees at 25 percent level for Delta Gamma_s**
- **Agrees at 70 percent level for D+/D0**

Precision mostly limited by theory

- **Can be improved by about a factor of two by sum rules**
- **Can be improved considerably by lattice**
- **Can extended to more hadron systems**

Can we make some generic statements about the remaining possible size of violations of QHD violations and about its consequences?

Try a parametrisation of potential QHD violations

HQE is actually an expansion in 1/ momentum release $\sqrt{M_i^2 - M_f^2}$

For the case of Bs decays

$$M_{B_s^0} - M_K - M_\pi = 4.73\,\text{GeV}\,,$$

$$M_{B_s^0} - M_{D_s^+} - M_\pi = 3.26\,\text{GeV}\,,$$

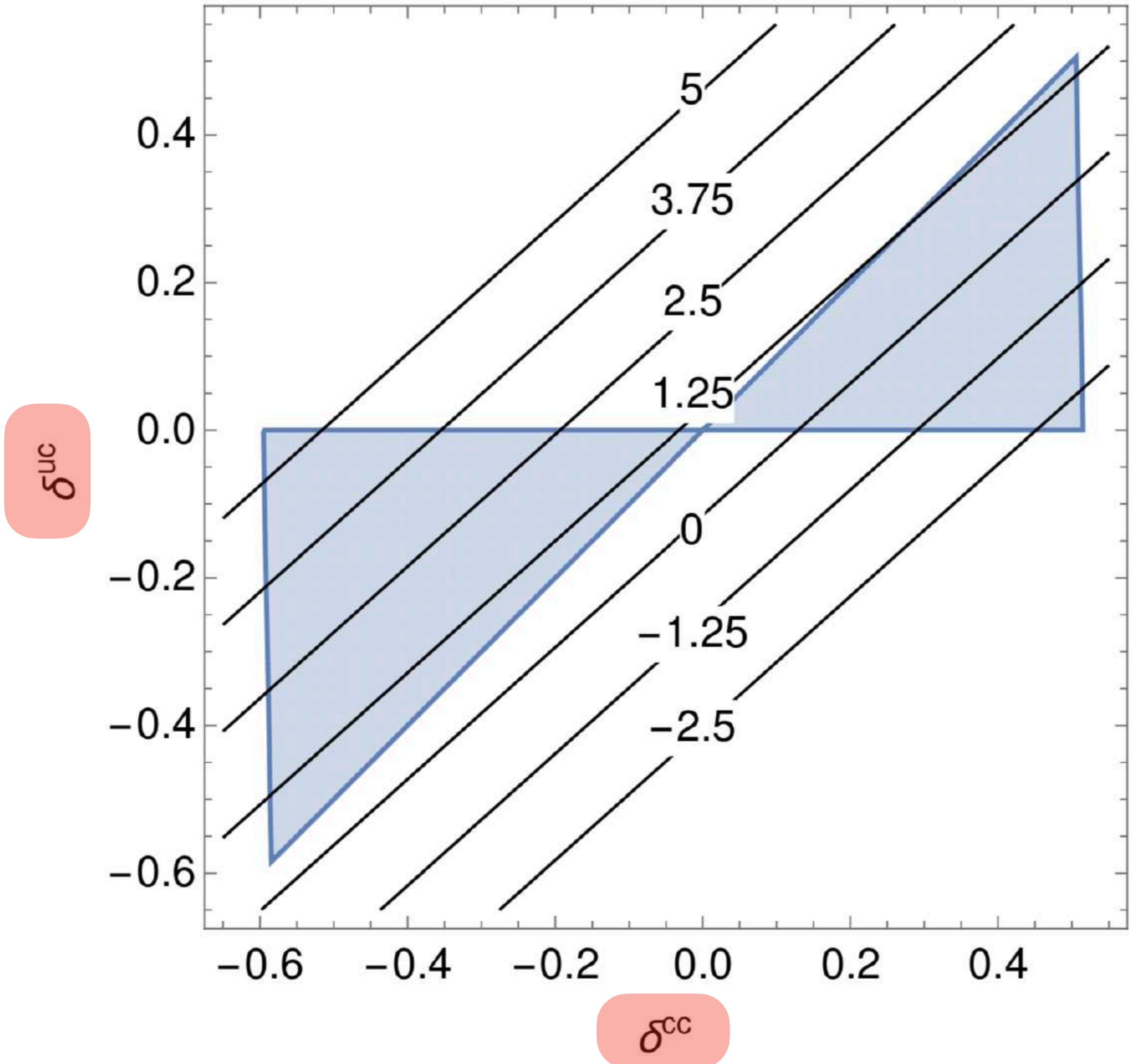
$$M_{B_s^0} - 2M_{D_s^{(*)+}} = 1.43(1.15)\,\text{GeV}\,.$$

Seems to be worse for heavier final states, model:

$$\Gamma_{12}^{s,cc} \to \Gamma_{12}^{s,cc}(1 + 4\delta)\,,$$

$$\Gamma_{12}^{s,uc} \to \Gamma_{12}^{s,uc}(1 + \delta)\,,$$

$$\Gamma_{12}^{s,uu} \to \Gamma_{12}^{s,uu}(1 + 0\delta)\,.$$

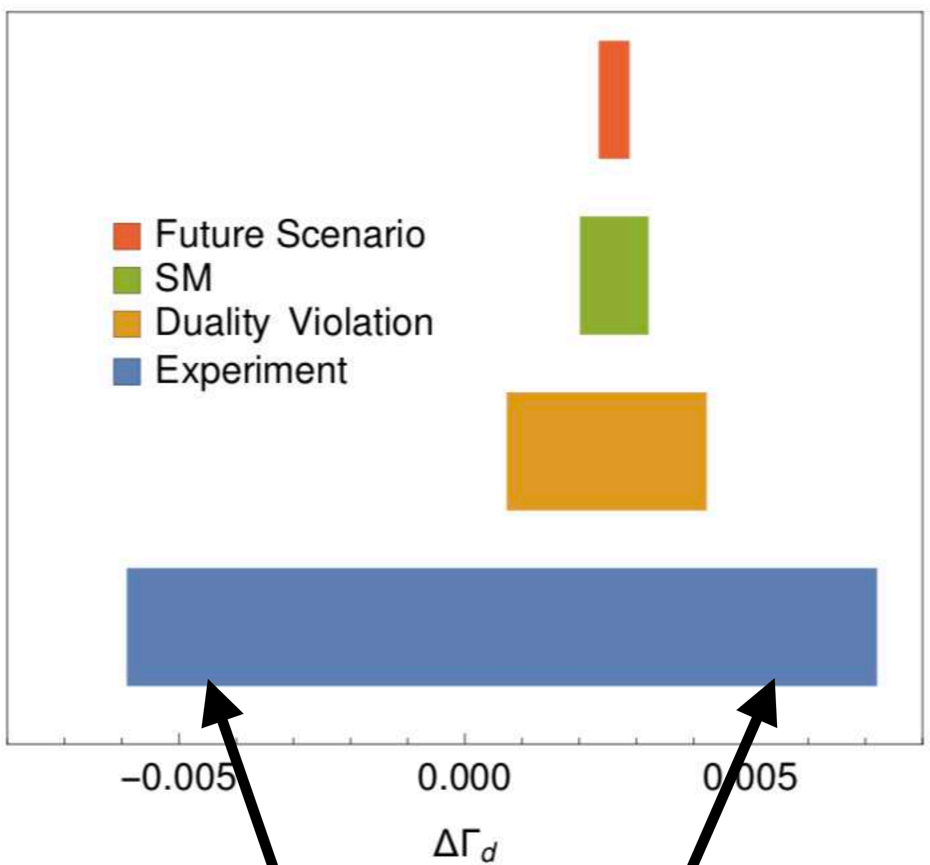


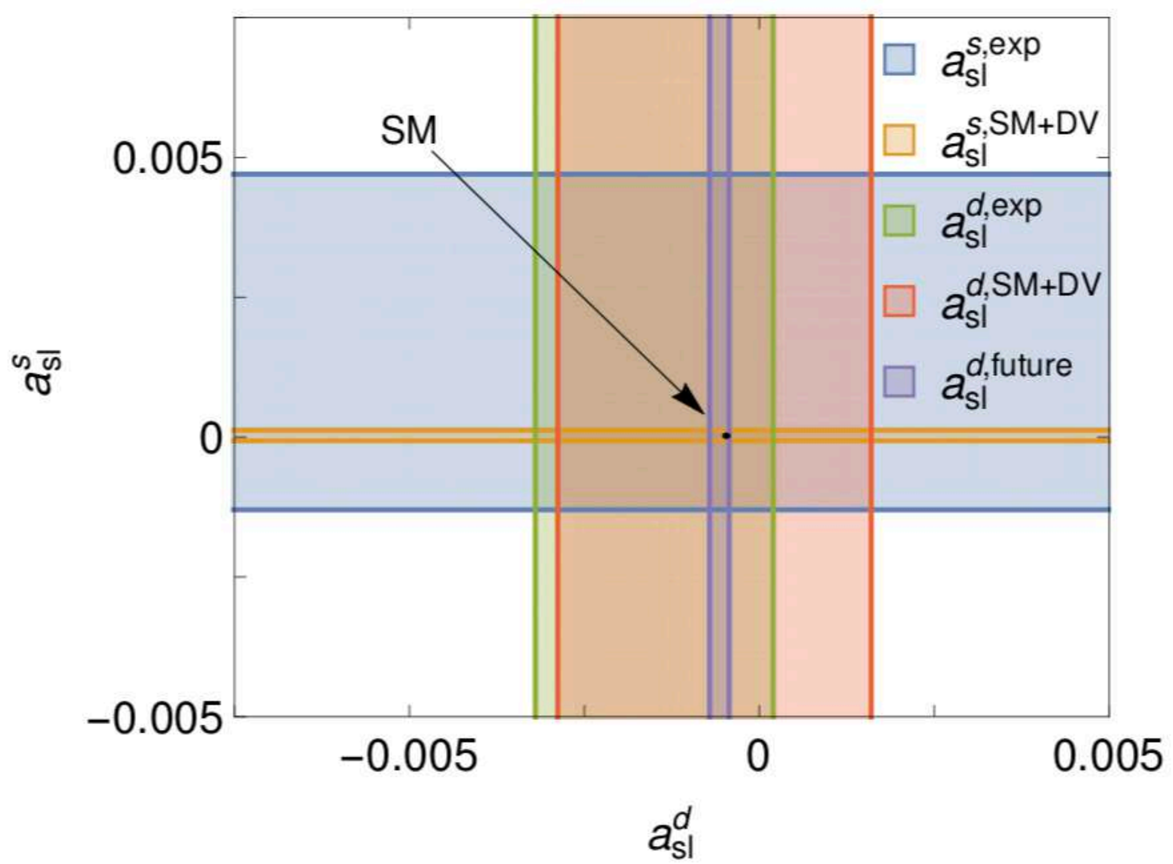
1603.07770

$\delta \in [-0.066, +0.046]$

Try a parametrisation of potential QHD violations

Exactly the same diagrams contribute to semi-leptonic asymmetries and Delta Gamma_D => consequences for BSM searches



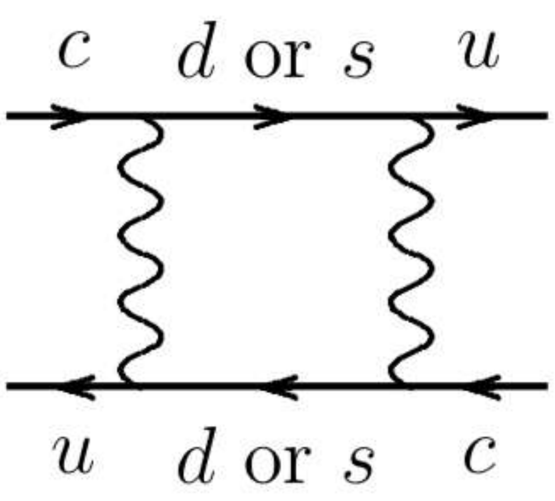


Any measurement outside the orange region cannot be due to duality violations

Charm mixing

Naive HQE estimate deviates by 10^4 from Exp

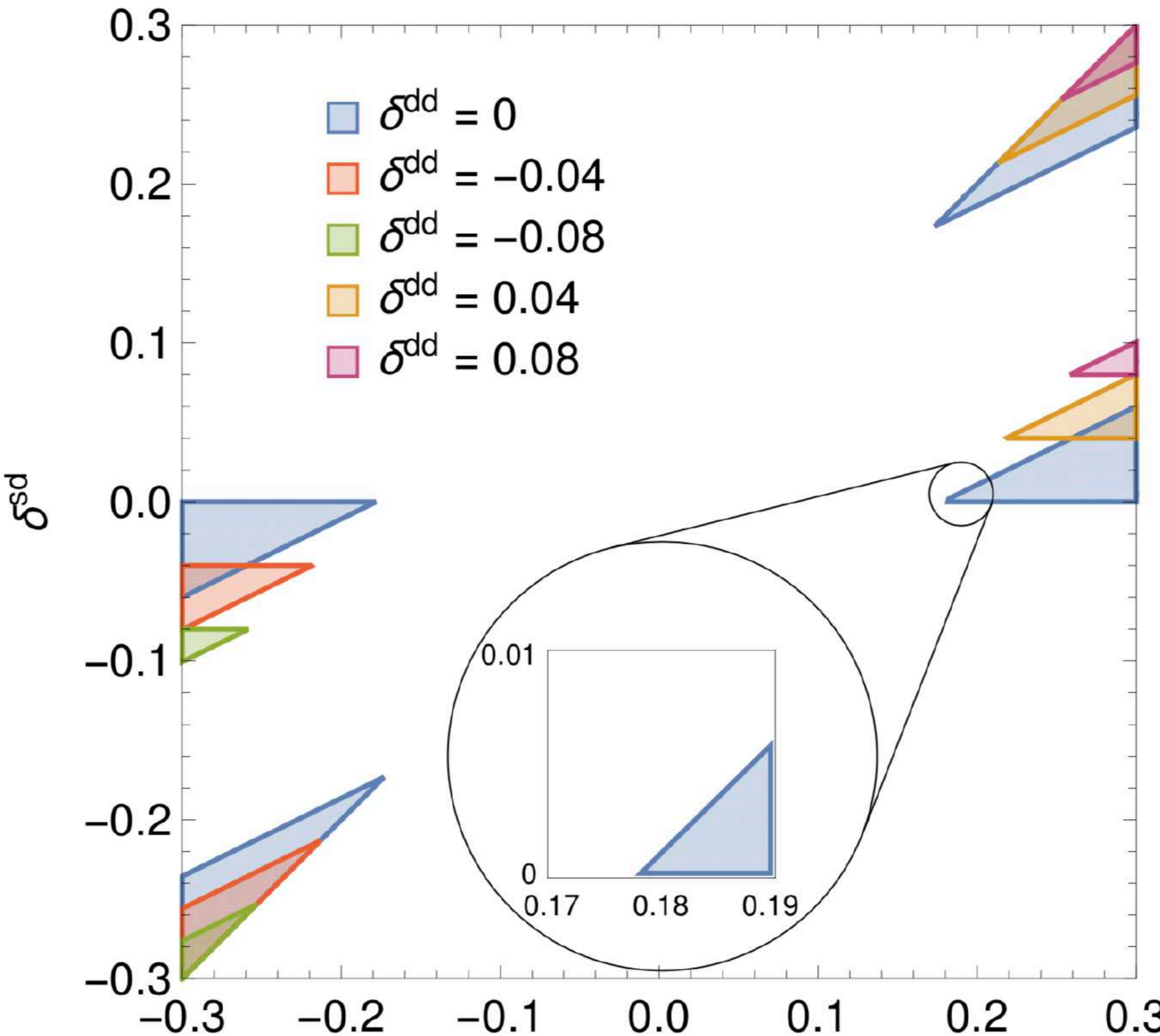
due to severe GIM cancellation of 3 contributions that are individually 5 times larger than experiment



20% of deviation from HQE expectation sufficient to explain experiment! Not 1000000%

So far no proof for this possibility, but many doable ideas around to test that idea

Try a parametrisation of potential QHD violations



$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1 + 4\delta) \ ,$$

$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1 + \delta) \ ,$$

$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1 + 0\delta) \ ,$$

As naively expected: 20% of QHD violations might be sufficient to explain discrepancy between HQE and experiment

HQE - Conclusions

1. Total inclusive decay rates
   HQE works perfect for Bs/Bd sub-percent level
   HQE works well for B+/Bd 2 percent level
   HQE works well for Lambda_b/Bd 5 percent level
   Indication HQE works for D+/D0! 70% level
   Much more work has to be done for improving theoretical precision and extending studies to b-baryons, more charm mesons and more charmed baryons

2. Inclusive decay rates, like Gamma_12 of neutral B-mesons
   HQE works well for Bs 25 per cent level
   Much more work has to be done for improving theoretical precision and experimentally Delta Gamma_d and the semi-leptonic CP asymmetries have to be measured

3. Semileptonic-branching ratio, B_sl and determination of V_cb and V_ub.
   agrees well on the per cent level
   a kind of discrepancy between exclusive inclusive determinations
   exclusive values point towards further problems

Duality - Conclusions

1. Comparison of Experiment and theory show no sign of duality violations in a number of very different observables
   Total inclusive decay rates of heavy hadrons
   inclusive decay rates of heavy hadron
   hadronic tau decays
   e^+-e^- ,....
   Much more work can be done for improving theory precision

2. Theoretical approaches like SV-limit, t'Hooft model studies gave no indication for sizeable violation of quark-hadron duality

3. Large duality violating effects in the b-system clearly ruled out

4. Duality violation in the charm system as low as 20% could be responsible for explaining discrepancy between HQE predictions for D-mixing and experiment - should be further investigated!

5. Considerable phenomenological progress => Time to revisit theoretical studies of duality and its violations?